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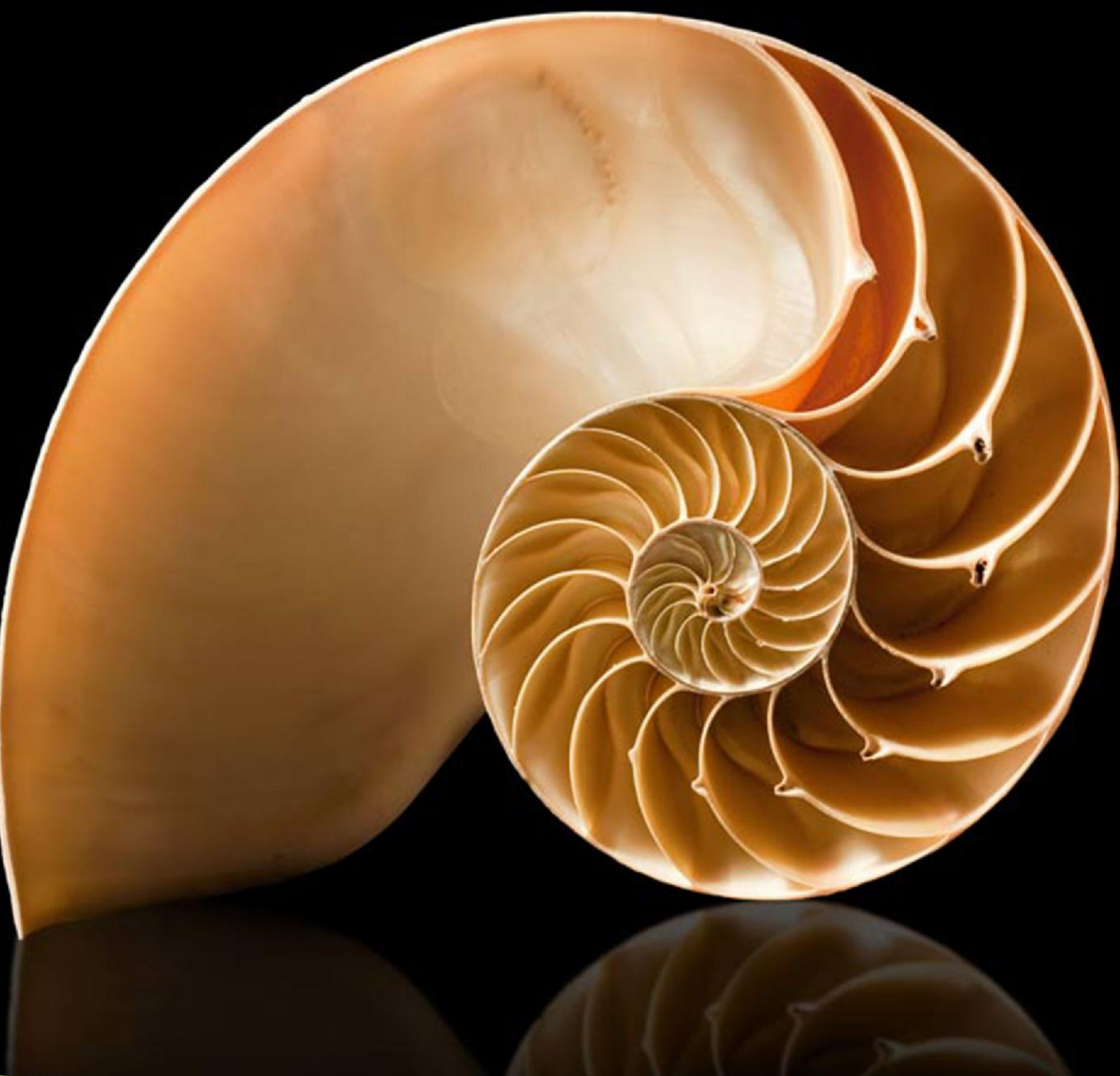
Cambridge International AS & A Level

# MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS



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# Appendix A

# Answers

1. 9709_s20_MS_21 Q: 2

| | | |
|---|---|---|
| | Substitute $x = 2$ and equate to zero | **M1** |
| | Substitute $x = -\frac{1}{2}$ and equate to zero | **M1** |
| | Obtain $4a + b + 66 = 0$ and $\frac{1}{4}a + b - \frac{21}{4} = 0$ or equivalents | **A1** |
| | Solve a relevant pair of linear simultaneous equations (Dependent on at least one M mark) | **DM1** |
| | Obtain $a = -19$, $b = 10$ | **A1** |
| | | **5** |

2. 9709_s20_MS_21 Q: 4

| | | |
|---|---|---|
| (a) | Draw two V-shaped graphs with one vertex on negative $x$-axis and one vertex on positive $x$-axis | **M1** |
| | Draw correct graphs related correctly to each other | **A1** |
| | State correct coordinates $-\frac{2}{3}a$, $2a$, $\frac{4}{3}a$, $4a$ | **A1** |
| | | **3** |
| (b) | Solve linear equation with signs of $3x$ different or solve non-modulus equation $(3x + 2a)^2 = (3x - 4a)^2$ | **M1** |
| | Obtain $x = \frac{1}{3}a$ | **A1** |
| | Obtain $y = 3a$ | **A1** |
| | | **3** |
| (c) | State $x < \frac{1}{3}a$ (**FT** from part **(b)**) | **B1FT** |
| | | **1** |

3. 9709_s20_MS_22 Q: 5

| | | |
|---|---|---|
| (a) | Draw V-shaped graph with vertex on positive $x$-axis | **B1** |
| | Draw (more or less) correct graph of $y = 3x + 5$ | **B1** |
| | | **2** |
| (b) | State equation $3x + 5 = -(2x - 3)$ or corresponding inequality | **B1** |
| | Attempt solution of linear equation / inequality where signs of $3x$ and $2x$ are different | **M1** |
| | State answer $x < -\frac{2}{5}$ | **A1** |
| | **Alternative method for question 5(b)** | |
| | Square both sides of equation / inequality and attempt solution of 3-term quadratic equation / inequality | **M1** |
| | Obtain (eventually) only $-\frac{2}{5}$ | **A1** |
| | State answer $x < -\frac{2}{5}$ | **A1** |
| | | **3** |

4. 9709_w20_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use correct logarithm property to simplify left-hand side | M1 | Or equivalent method |
| | Use correct process to obtain equation without logarithms | M1 | |
| | Obtain $\frac{2x+1}{x-3} = e^2$ | A1 | OE |
| | Obtain $x = \frac{3e^2+1}{e^2-2}$ | A1 | OE |
| | | 4 | |

5. 9709_w20_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Substitute $x = -2$ and equate to zero | *M1 | |
| | Substitute $x = 2$ and equate to 72 | *M1 | |
| | Obtain $4a - 2b + 8 = 0$ and $4a + 2b - 48 = 0$ or equivalents | A1 | |
| | Solve a pair of relevant linear simultaneous equations | DM1 | Dependent at least one M mark |
| | Obtain $a = 5$, $b = 14$ | A1 | |
| | | 5 | |

6. 9709_w20_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | State or imply non-modulus equation $(2x-5)^2=(x+6)^2$ or pair of linear equations | B1 | |
| | Attempt solution of 3-term quadratic equation or of pair of linear equations | M1 | |
| | Obtain $-\frac{1}{3}$ and 11 | A1 | |
| | | 3 | |
| (b) | Apply logarithms and use power law for $2^{-y}=k$ where $k>0$ from **(a)** | M1 | |
| | Obtain $-3.46$ | A1 | AWRT |
| | | 2 | |

7. 9709_w20_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Draw V-shaped graph with vertex on positive *x*-axis | B1 | |
| | Draw straight line graph correctly positioned with greater gradient | B1 | |
| | | 2 | |
| (b) | Solve linear equation with signs of $\frac{1}{2}x$ and $\frac{3}{2}x$ different **or** solve non-modulus equation $\left(\frac{1}{2}x-a\right)^2=\left(\frac{3}{2}x-\frac{1}{2}a\right)^2$ to obtain $x=$ | M1 | |
| | Obtain $x=\frac{3}{4}a$ | A1 | |
| | Obtain $y=\frac{5}{8}a$ | A1 | And no other point |
| | | 3 | |
| (c) | State $x<\frac{3}{4}a$ | B1 FT | Following *their* (single) *x*-coordinate from part **(b)** |
| | | 1 | |

8. 9709_m19_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Solve non-modular equation $(2x+3)^2=(2x-1)^2$ or linear equation with signs of $2x$ different | M1 | |
| | Obtain $x=-\frac{1}{2}$ | A1 | |
| | Substitute negative value into expression and show correct evaluation of modulus at least once | M1 | |
| | Obtain $5-3=2$ with no errors seen | A1 | |
| | | 4 | |

9. 9709_m19_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Carry out division at least as far as $2x^2 + kx$ | **M1** | |
| | Obtain quotient $2x^2 + 3x + 4$ | **A1** | |
| | Confirm remainder is 5 | **A1** | Answer given; necessary detail needed |
| | | **3** | |
| (ii) | State or imply equation is $(2x+1)(2x^2+3x+4)=0$ | **B1** | FT their quotient from part **(i)** |
| | Calculate discriminant of 3-term quadratic expression or equivalent | **M1** | |
| | Obtain –23 or equiv and conclude appropriately | **A1** | |
| | | **3** | |

10. 9709_s19_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply non-modular inequality $(3x-5)^2 < (x+3)^2$ or corresponding equation or pair of different linear equations/inequalities | **B1** | SC: Allow B1 for $x < 4$ from only one linear inequality |
| | Attempt solution of 3-term quadratic equation/inequality or of two different linear equations/inequalities | **M1** | For M1, must get as far as 2 critical values |
| | Obtain critical values $\frac{1}{2}$ and 4 | **A1** | |
| | State answer $\frac{1}{2} < x < 4$ or equivalent | **A1** | If given as 2 separate statements, condone omission of 'and' or $\cap$ but penalise inclusion of 'or' or $\cup$ |
| | | **4** | |
| (ii) | Attempt to find $n$ (not necessarily an integer so far) from $3^{0.1n} =$ or < *their* positive upper value from part **(i)** or $3^{0.1n+1} =$ or < 3 × *their* positive upper value from part **(i)** | **M1** | 0/2 for trial and improvement |
| | Conclude 12 | **A1** | |
| | | **2** | |

11. 9709_s19_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $x = 2$ and equate to zero | **M1** | Allow synthetic division for each– must result in an equation from each division |
| | Substitute $x = -1$ and equate to 27 | **M1** | Allow unsimplified |
| | Obtain $4a + 2b = -24$ and $a - b = 48$ or equivalents | **A1** | Allow one error in each equation |
| | Solve a relevant pair of simultaneous linear equations | **M1** | Dependent at least one M mark |
| | Obtain $a = 12$, $b = -36$ | **A1** | |
| | | **5** | |
| (ii) | Divide by $x - 2$ at least as far as the $x$ term to obtain $5x^2 + (their\ a + 10)x...$ | **M1** | For synthetic division need to see 5 and *their* $a + 10$ in the bottom line |
| | Obtain $5x^2 + 22x + 8$ | **A1** | |
| | Obtain $(x-2)(5x+2)(x+4)$ | **A1** | If solved using a calculator and then forming factors, must be correct for full marks |
| | | **3** | |

12. 9709_s19_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Substitute $-1$ into $p(x)$ and equate to zero | **M1** | Allow algebraic long division or the use of an identity with the remainder, in terms of *m* and *k*, equated to zero |
| | Obtain $-4+(k+1)+m+3k=0$ or equivalent | **A1** | |
| | Obtain $m=3-4k$ | **A1** | |
| | | **3** | |

13. 9709_s19_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply non-modular equation $(4+2x)^2=(3-5x)^2$ or pair of linear equations | **B1** | |
| | Attempt solution of 3-term quadratic eqn or pair of linear equations | **M1** | |
| | Obtain $-\frac{1}{7}$, $\frac{7}{3}$ | **A1** | **SC** B1 for $x=-\dfrac{1}{7}$ from one linear equation |
| | | **3** | |
| (ii) | Attempt correct process to solve $e^{3y}=k$ where $k>0$ from **(i)** | **M1** | |
| | Obtain 0.282 and no others | **A1** | |
| | | **2** | |

14. 9709_w19_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply non-modular inequality $(2x-7)^2<(2x-9)^2$ or corresponding equation or linear equation (with signs of $2x$ different) | **M1** | |
| | Obtain critical value 4 | **A1** | |
| | State $x<4$ only | **A1** | |
| | | **3** | |
| (ii) | Attempt to find *n* from $\ln n$ = *their* critical value from part **(i)** | **M1** | |
| | Obtain or imply $n<e^4$ and hence 54 | **A1** | |
| | | **2** | |

15. 9709_w19_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $x=2$, equate to zero and attempt solution | M1 | |
| | Obtain $a=4$ | A1 | |
| | | 2 | |
| (ii) | Divide by $x-2$ at least as far as the $x$ term | M1 | By inspection or use of identity |
| | Obtain $4x^2+12x+9$ | A1 | |
| | Conclude $(x-2)(2x+3)^2$ | A1 | Each factor must be simplified to integer form |
| | | 3 | |
| (iii) | Attempt correct process to solve $e^{\sqrt{y}}=k$ where $k>0$ | M1 | For $y=(\ln k)^2$ |
| | Obtain 0.48 and no others | A1 | AWRT |
| | | 2 | |

16. 9709_w19_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Divide at least as far as the $x$ term in the quotient | M1 | Allow use of $(x^2+2)(x^2+ax+b)+cx+d$ |
| | Obtain at least $x^2-3x$ | A1 | |
| | Obtain $x^2-3x+3$ and remainder 5 | A1 | |
| | | 3 | |

17. 9709_m18_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | *EITHER:*<br>State or imply non-modular inequality $(5x+2)^2>(4x+3)^2$ or corresponding equation or pair of linear equations | (B1 | |
| | Attempt solution of 3-term quadratic equation or of 2 linear equations | M1 | |
| | Obtain critical values $-\frac{5}{9}$ and 1 | A1 | And no others |
| | State answer $x<-\frac{5}{9}$, $x>1$ | A1) | |
| | *OR:*<br>Obtain critical value $x=1$ from graph, inspection, equation | (B1 | |
| | Obtain critical value $x=-\frac{5}{9}$ similarly | B2 | |
| | State answer $x<-\frac{5}{9}$, $x>1$ | B1) | |
| | | 4 | |

18. 9709_m18_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $x = -3$ and simplify | **M1** | |
| | Obtain $-108 + 36 + 87 - 15 = 0$ or equivalent and conclude | **A1** | |
| | | **2** | |
| (ii) | Attempt <u>either</u> division by $x + 3$ to reach at least partial quotient $4x^2 + kx$ <u>or</u> use of identity <u>or</u> inspection | **M1** | |
| | Obtain quotient $4x^2 - 8x - 5$ | **A1** | |
| | Conclude $(x + 3)(2x - 5)(2x + 1)$ | **A1** | |
| | | **3** | |
| (iii) | Identify $2^u = \frac{5}{2}$ | **B1** | Ignoring other values at this stage |
| | Apply logarithms and use power law for $2^u = c$ where $c > 0$ | **M1** | |
| | Obtain $u = 1.32$ | **A1** | And no other values |
| | | **3** | |

19. 9709_s18_MS_21 Q: 6

<table>
<tr><th></th><th>Answer</th><th>Mark</th><th>Partial Marks</th></tr>
<tr><td>(i)</td><td>Substitute $x=-2$ and equate to zero</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $-8+4a-28+a+1=0$ or equivalent and hence $a=7$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Attempt <u>either</u> division by $x+2$ and reach partial quotient $x^2+kx$, where $k$ is numeric <u>or</u> use of identity <u>or</u> inspection <u>or</u> synthetic division</td><td>M1</td><td>Synthetic division:<table><tr><td>–2</td><td>1</td><td>7</td><td>14</td><td>8</td></tr><tr><td></td><td></td><td>–2</td><td>–10</td><td>–8</td></tr><tr><td></td><td>1</td><td>5</td><td>4</td><td>0</td></tr></table></td></tr>
<tr><td></td><td>Obtain quotient $x^2+5x+4$ soi</td><td>A1</td><td></td></tr>
<tr><td></td><td>Conclude with $(x+1)(x+2)(x+4)$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
<tr><td>(ii)</td><td><u>Either</u></td><td></td><td></td></tr>
<tr><td></td><td>State $(2x+1)(2x+2)(2x+4)=3(x+1)(x+2)(x+4)$</td><td>M1</td><td>Following their complete factorised form</td></tr>
<tr><td></td><td>Obtain $x=-1$ and $x=-2$</td><td>A1</td><td>Calculator not permitted so necessary detail needed</td></tr>
<tr><td></td><td>Cancel common factors to obtain linear equation or factorise to find corresponding factor</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $x=\frac{8}{5}$ or equivalent</td><td>A1</td><td></td></tr>
<tr><td></td><td><u>Or</u></td><td></td><td></td></tr>
<tr><td></td><td>State $(2x+1)(2x+2)(2x+4)=3(x+1)(x+2)(x+4)$ or<br>$(2x)^3+7(2x)^2+14(2x)+8=3(x^3+7x^2+14x+8)$</td><td>M1</td><td>Following their completed factorised form,<br>Must see $8x^3$ and $28x^2$ if using second statement without bracketed terms in $2x$</td></tr>
<tr><td></td><td>Expand and simplify to obtain $5x^3+7x^2-14x-16=0$</td><td>A1</td><td>Must be equated to 0 for A1</td></tr>
<tr><td></td><td>Attempt complete factorisation of cubic with leading term $5x^3$ (may make use of synthetic division)</td><td>M1</td><td>Synthetic division:<table><tr><td>–2</td><td>5</td><td>7</td><td>–14</td><td>–16</td></tr><tr><td></td><td></td><td>–10</td><td>6</td><td>16</td></tr><tr><td></td><td>5</td><td>–3</td><td>–8</td><td>0</td></tr></table></td></tr>
<tr><td></td><td>Obtain $(x+1)(x+2)(5x-8)=0$ and conclude $x=-1$, $x=-2$, $x=\frac{8}{5}$</td><td>A1</td><td>Calculator not permitted so necessary detail needed</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

20. 9709_s18_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Either | | |
| | State or imply non-modular inequality $(3x-2)^2 < (x+5)^2$ or corresponding equation or pair of linear equations | B1 | |
| | Attempt solution of 3-term quadratic equation or of 2 linear equations | M1 | |
| | Obtain critical values $-\frac{3}{4}$ and $\frac{7}{2}$ | A1 | |
| | State answer $-\frac{3}{4} < x < \frac{7}{2}$ | A1 | |
| | Or | | |
| | Obtain critical value $\frac{7}{2}$ from graph, inspection, equation | B1 | |
| | Obtain critical value $-\frac{3}{4}$ similarly | B2 | |
| | State answer $-\frac{3}{4} < x < \frac{7}{2}$ | B1 | |
| | | 4 | |

21. 9709_s18_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Carry out division and reach at least partial quotient of form $x^2 + kx$ | M1 | |
| | Obtain quotient $x^2 - 2x + 2$ | A1 | |
| | Obtain remainder 1 | A1 | AG; necessary detail needed and all correct |
| | | 3 | |
| (ii) | State equation as $(x^2+6)(x^2-2x+2) = 0$ | B1 FT | Following their 3-term quotient from part (i) |
| | Calculate discriminant of 3-term quadratic or equivalent | M1 | |
| | Obtain $-4$ and state no root, also referring to no root from $x^2+6$ factor | A1 | AG; necessary detail needed |
| | | 3 | |

22. 9709_w18_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Either | | |
| | State or imply non-modular inequality $(3x-5)^2 < 4x^2$ or corresponding equation or pair of linear equations | B1 | SC: Common error $(3x-5)^2 < 2x^2$ |
| | Attempt solution of 3-term quadratic equation or solution of 2 linear equations | M1 | |
| | Obtain critical values 1 and 5 | A1 | Critical values $\frac{15\pm5\sqrt{2}}{7}$ or 3.15, 1.13 allow B1 |
| | State correct answer $1 < x < 5$ | A1 | $\frac{15-5\sqrt{2}}{7} < x < \frac{15+5\sqrt{2}}{7}$ or $1.13 < x < 3.15$ B1<br>Max 2/4<br>Allow M1 for $(7x\pm5)(x\pm5)$ |
| | Or | | |
| | Obtain $x = 5$ by solving linear equation or inequality or from graphical method or inspection | B1 | Allow B1 for 5 seen, maybe in an inequality |
| | Obtain $x = 1$ similarly | B2 | Allow B2 for 1 seen, maybe in an inequality |
| | State correct answer $1 < x < 5$ | B1 | |
| | | 4 | |

23. 9709_m17_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $x = -2$ and equate to zero | M1 | |
| | Substitute $x = 2$ and equate to 28 | M1 | |
| | Obtain $-9a + 4b + 34 = 0$ and $7a + 4b - 62 = 0$ or equivalents | A1 | |
| | Solve a relevant pair of simultaneous equations for $a$ or $b$ | M1 | |
| | Obtain $a = 6$, $b = 5$ | A1 | |
| | **Total:** | **5** | |
| (ii) | Divide by $x + 2$, or equivalent, at least as far as $k_1x^2 + k_2x$ | M1 | |
| | Obtain $6x^2 - 7x - 3$ | A1 | |
| | Obtain $(x+2)(3x+1)(3x-3)$ | A1 | |
| | **Total:** | **3** | |
| (iii) | Refer to, or clearly imply, fact that $2^y$ is positive | M1 | |
| | State one | A1√ | following 3 linear factors from part **(ii)** |
| | **Total:** | **2** | |

24. 9709_s17_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | State or imply non-modulus inequality $(4-x)^2 \leqslant (3-2x)^2$ or corresponding equation, pair of linear equations or linear inequalities | M1 | |
| | Attempt solution of 3-term quadratic equation, of two linear equations or of two linear inequalities | M1 | |
| | Obtain critical values $-1$ and $\frac{7}{3}$ | A1 | SR Allow **B1** for $x \leqslant -1$ only or $x \geqslant \frac{7}{3}$ only if first **M1** is not given |
| | State answer $x \leqslant -1, x \geqslant \frac{7}{3}$ | A1 | Do not accept $\frac{7}{3} \leqslant x \leqslant -1$ or $-1 \geqslant x \geqslant \frac{7}{3}$ for **A1** |
| | **Total:** | 4 | |

25. 9709_s17_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | State or imply non-modulus equation $(x+a)^2 = (2x-5a)^2$ or pair of linear equations | B1 | SR **B1** for $x = 6a$ |
| | Attempt solution of quadratic equation or of pair of linear equations | M1 | Allow **M1** if $\frac{4}{3}$ and 6 seen |
| | Obtain, as final answers, $6a$ and $\frac{4}{3}a$ | A1 | |
| | **Total:** | 3 | |

26. 9709_s17_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Evaluate expression when $x=-2$ | M1 | |
| | Obtain 0 with all necessary detail present | A1 | Use of $f(x)=(x+2)(ax^2+bx+c)$ to find $a$, $b$ and $c$, allow **M1 A0**<br>Use of $f(x)=(x+2)(ax^2+bx+c)+d$ to find $a$, $b$ and $c$, and show $d=0$, allow **M1 A1** |
| | Carry out division, or equivalent, at least as far as $x^2$ and $x$ terms in quotient | M1 | |
| | Obtain $6x^2+x-35$ | A1 | |
| | Obtain factorised expression $(x+2)(2x+5)(3x-7)$ | A1 | |
| | **Total:** | 5 | |
| (ii) | State or imply substitution $x=\frac{1}{y}$ or equivalent | M1 | |
| | Obtain $-\frac{1}{2}$, $-\frac{2}{5}$, $\frac{3}{7}$ | A1 | |
| | **Total:** | 2 | |

27. 9709_w17_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $x = -2$ and equate to zero | *M1 | |
| | Substitute $x = \frac{1}{2}$ and equate to 40 | *M1 | |
| | Obtain $-8a + 4b - 64 = 0$ and $\frac{1}{8}a + \frac{1}{4}b = \frac{23}{2}$ or equivalents | A1 | |
| | Solve a pair of simultaneous equations for $a$ or for $b$ | DM1 | Needs at least one of the two previous M marks |
| | Obtain $a = 12$ and $b = 40$ | A1 | |
| | | 5 | |
| (ii) | Attempt division by $(x+2)$ or inspection at least as far as $kx^2 + mx$ | M1 | |
| | Obtain $12x^2 + 16x + 5$ | A1 | |
| | Conclude $(x+2)(2x+1)(6x+5)$ | A1 | |
| | | 3 | |

28. 9709_w17_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Solve 3-term quadratic equation or a pair of linear equations | M1 | For **M1**, must square both sides when attempting a quadratic equation |
| | Obtain $x = -5$ and $x = 3$ | A1 | |
| | Substitute (at least) one value of $x$ (less than 4) into $\lvert x+4\rvert - \lvert x-4\rvert$, showing correct evaluation of modulus and producing only one answer in each case | M1 | |
| | Obtain $-8$ and 6 and no others | A1 | |
| | | 4 | |

29. 9709_w17_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $x=-3$ into either $\text{p}(x)$ or $\text{q}(x)$ and equate to zero ( may be implied) | **M1** | Allow long division, but the remainder needs to be independent of $x$ |
| | Obtain $a=-11$ | **A1** | |
| | Obtain $b=-8$ | **A1** | |
| | | **3** | |
| (ii) | Divide $x+3$ into expression for $\text{q}(x)-\text{p}(x)$ ( may be a four term cubic equation), or Obtain a 3 term cubic equation by subtraction | ***M1** | Allow *M1 for their $x^3+3x+36$, but must have integer values for $a$ and $b$ |
| | Obtain $x^2-3x+12$ or $x^2-2x-5$ and $2x^2-5x+7$ | **A1** | |
| | Apply discriminant to quadratic factor of $\text{q}(x)-\text{p}(x)$ or equivalent | **DM1** | dep on *M |
| | Obtain $-39$ or equivalent and conclude appropriately | **A1** | |
| | | **4** | |

30. 9709_m16_MS_22 Q: 1

Attempt division at least as far as quotient $2x^2+kx$ **M1**
Obtain quotient $2x^2-x+2$ **A1**
Obtain remainder 6 **A1** [3]

Special case: Use of Remainder Theorem to give 6 **B1**

31. 9709_m16_MS_22 Q: 2

Either State or imply non-modular inequality $(x-5)^2<(2x+3)^2$ or corresponding pair of linear equations **B1**
Attempt solution of 3-term quadratic equation or of 2 linear equations **M1**
Obtain critical values $-8$ and $\frac{2}{3}$ **A1**
State answer $x<-8,\quad x>\frac{2}{3}$ **A1**

Or Obtain critical value $-8$ from graphical method, inspection, equation **B1**
Obtain critical value $\frac{2}{3}$ similarly **B2**
State answer $x<-8,\quad x>\frac{2}{3}$ **B1** [4]

32. 9709_s16_MS_21 Q: 4

| | | | |
|---|---|---|---|
| **(i)** | Carry out division, or equivalent, at least as far as $8x^2 + kx$ | **M1** | |
| | Obtain correct quotient $8x^2 + 14x - 15$ | **A1** | |
| | Confirm remainder is 5 | **A1** | [3] |
| **(ii)** | State or imply expression is $(x+2)$(...their quadratic quotient...) | **B1**√ | |
| | Attempt factorisation of their quadratic quotient | **M1** | |
| | Obtain $(x+2)(2x+5)(4x-3)$ | **A1** | [3] |
| **(iii)** | State $\pm\frac{3}{4}$ and no others, following their 3 linear factors | **B1**√ | [1] |

---

33. 9709_s16_MS_22 Q: 2

| | | | |
|---|---|---|---|
| **(i)** | Carry out division, or equivalent, at least as far as quotient $2x + k$ | **M1** | |
| | Obtain quotient $2x - 3$ | **A1** | |
| | Obtain remainder $-25x + 18$ | **A1** | [3] |
| **(ii)** | Subtract remainder of form $ax + b$ ($ab \neq 0$) from $2x^3 - 7x^2 - 9x + 3$ or multiply their quotient by $x^2 - 2x + 5$ | **M1** | |
| | Obtain $p = 16$ and $q = -15$ | **A1** | [2] |

---

34. 9709_s16_MS_22 Q: 3

| | | | |
|---|---|---|---|
| **(i)** | State or imply non-modular equation $(3u+1)^2 = (2u-5)^2$ or corresponding pair of linear equations | **B1** | |
| | Attempt solution of 3-term quadratic equation or of 2 linear equations | **M1** | |
| | Obtain $-6$ and $\frac{4}{5}$ | **A1** | [3] |
| **(ii)** | Evaluate $\tan^{-1}\frac{1}{k}$ for at least one of their solutions $k$ from part **(i)** | **M1** | |
| | Obtain 0.896 | **A1** | [2] |

---

35. 9709_w16_MS_22 Q: 1

| | | | | |
|---|---|---|---|---|
| | State non-modulus equation $(0.4x - 0.8)^2 = 4$ or equivalent or corresponding pair of linear equations | **B1** | | SR One solution only – B1 |
| | Solve 3-term quadratic equation or pair of linear equations | **M1** | | Must see some evidence of attempt to solve the quadratic for M1 for at least one value of $x$ For a pair of linear equations, there must be a sign difference |
| | Obtain $-3$ and $7$ | **A1** | [3] | If extra solutions are given then A0 |

---

36. 9709_w16_MS_22 Q: 4

| | | | | |
|---|---|---|---|---|
| (i) | Substitute $x=-1$ and simplify | M1 | | Allow attempt at long division , must get down to a remainder<br>Allow M1 if at least 2 numerical values of $a$ are used<br>May equate to $(x+1)(Ax^2+Bx+C)+R$ - allow M1 if they get as far as finding $R$ |
| | Obtain $-4+a-a+4=0$ and conclude appropriately | A1 | [2] | Must have a conclusion - allow 'hence shown', or made a statement of intent at the start of the question |
| (ii) | Substitute $x=2$ and equate to $-42$ and attempt to solve | M1 | | May equate to $(x-2)(Ax^2+Bx+C)$, must have a complete method to get as far as $a=$ ... to obtain M1 |
| | Obtain $a=-13$ | A1 | [2] | |
| (iii) | Divide $\text{p}(x)$ with their $a$ at least as far as $4x^2+kx$ | M1 | | |
| | Obtain $4x^2-17x+4$ | A1 | | |
| | Obtain $(x+1)(4x-1)(x-4)$ or equivalent if $x^2$ already involved | A1 | | If $(x+1)(4x-1)(x-4)$ seen with no evidence of long division then allow the marks |
| | Obtain $(x^2+1)(2x-1)(2x+1)(x-2)(x+2)$ | A1 | [4] | |

37. 9709_w16_MS_23 Q: 4

| | | | |
|---|---|---|---|
| (i) | Substitute $x=\frac{1}{2}$ and equate to zero<br>Obtain $a=2$ | M1<br>A1 | [2] |
| (ii) | Divide by $2x-1$ at least as far as $x^2+kx$<br>Obtain quotient $x^2+2x+5$<br>Calculate discriminant of 3-term quadratic expression or equivalent<br>Obtain $-16$ and conclude appropriately | M1<br>A1<br>M1<br>A1 | [4] |
| (iii) | Use logarithms with power law shown in solving $6^y=\frac{1}{2}$<br>Obtain $-0.387$ | M1<br>A1 | [2] |

38. 9709_s15_MS_21 Q: 4

| | | | |
|---|---|---|---|
| (i) | Substitute $x=-2$ in $f(x)$ and equate to zero to obtain $-8+4a+b=0$ or equiv | B1 | |
| | Substitute $x=-1$ in $g(x)$ and equate to $-18$ | M1 | |
| | Obtain $-1+b-a=-18$ or equivalent | A1 | |
| | Solve a pair of linear equations for $a$ or $b$ | DM1 | |
| | Obtain $a=5$, $b=-12$ | A1 | [5] |
| (ii) | Simplify $g(x)-f(x)$ to obtain form $kx^2+c$ where $k<0$ | M1 | |
| | Obtain $-17x^2+7$ and state 7, following their value of $c$ | A1√ | [2] |

39. 9709_s15_MS_22 Q: 2

| | | | |
|---|---|---|---|
| (i) | Substitute $x=-2$ into expression and equate to zero | M1 | |
| | Obtain $-32+4a+2(a+1)-18=0$ or equivalent | A1 | |
| | Obtain $a=8$ | A1 | [3] |
| (ii) | Attempt to find quadratic factor by division, inspection, … | M1 | |
| | Obtain $4x^2-9$ | A1 | |
| | State $(x+2)(2x-3)(2x+3)$ | A1 | [3] |

40. 9709_w15_MS_21 Q: 6

| | | | | |
|---|---|---|---|---|
| (i) | Carry out division at least as far as quotient $x^2+kx$ | | **M1** | |
| | Obtain partial quotient $x^2+2x$ | | **A1** | |
| | Obtain quotient $x^2+2x+1$ with no errors seen | | **A1** | |
| | Obtain remainder $5x+2$ | | **A1** | [4] |
| (ii) | Either | Carry out calculation involving $12x+6$ and their remainder $ax+b$ | **M1** | |
| | | Obtain $p=7, q=4$ | **A1** | |
| | Or | Multiply $x^2-x+4$ by their three-term quadratic quotient | **M1** | |
| | | Obtain $p=7, q=4$ | **A1** | [2] |
| (iii) | Show that discriminant of $x^2-x+4$ is negative | | **B1** | |
| | Form equation $(x^2-x+4)(x^2+2x+1)=0$ and attempt solution | | **M1** | |
| | Show that $x^2+2x+1=0$ gives one root $x=-1$ | | **A1** | [3] |

41. 9709_w15_MS_23 Q: 4

| | | | |
|---|---|---|---|
| (i) | Attempt division, or equivalent, at least as far as quotient $3x^2+kx$ | **M1** | |
| | Obtain partial quotient $3x^2+11x$ | **A1** | |
| | Obtain complete quotient $3x^2+11x+20$ with no errors seen | **A1** | |
| | Confirm remainder is 39 | **B1** | [4] |
| (ii) | State or imply $(x-2)(3x^2+11x+20)=0$ | **B1** | |
| | Calculate discriminant of quadratic factor or equivalent | **M1** | |
| | Obtain $-119$ or equivalent and confirm only one real root | **A1** | [3] |

42. 9709_s20_MS_21 Q: 1

| | | |
|---|---|---|
| | Use correct logarithm property to produce one term on LHS | **M1** |
| | Use correct process to obtain equation without logarithms | **M1** |
| | Obtain $\frac{x+1}{x}=4$ or equivalent and hence $x=\frac{1}{3}$ | **A1** |
| | | **3** |

43. 9709_s20_MS_22 Q: 1

| | | |
|---|---|---|
| | Apply logarithms to both sides and apply power law at least once | **M1** |
| | Rearrange to the form $y=\frac{3\ln 9}{\ln 2}x$ OE | **A1** |
| | Obtain $k=9.51$ | **A1** |
| | | **3** |

44. 9709_s20_MS_22 Q: 4

| | | |
|---|---|---|
| | State or imply equation is $\ln y=\ln A-2p\ln x$ | **B1** |
| | Equate gradient of line to $-2p$ | **M1** |
| | Obtain $-2p=-2.6$ and hence $p=1.3$ | **A1** |
| | Substitute appropriate values to find $\ln A$ | **M1** |
| | Obtain $\ln A=1.252$ and hence $A=3.5$ | **A1** |
| | **Alternative method for question 4** | |
| | State or imply equation is $\ln y=\ln A-2p\ln x$ | **B1** |
| | Substitute given coordinates to obtain 2 simultaneous equations and solve to obtain $3.5p$ | **M1** |
| | Obtain $3.5p=4.55$ and hence $p=1.3$ | **A1** |
| | Substitute appropriate values to find $\ln A$ | **M1** |
| | Obtain $\ln A=1.252$ and hence $A=3.5$ | **A1** |
| | | **5** |

45. 9709_w20_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use $2^{3x+2}=4\times 2^{3x}$ | **B1** | OE |
| | Solve equation for $2^{3x}$ | **M1** | |
| | Obtain $2^{3x}=43$ | **A1** | |
| | Apply logarithms and use power law for $2^{3x}=k$ where $k>0$ | **M1** | |
| | Obtain 1.809 | **A1** | AWRT |
| | | **5** | |

46. 9709_m19_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | State or imply equation is $\ln y = \ln A + px + p$ | B1 | |
| | Equate gradient of line to $p$ | M1 | |
| | Obtain $p = 0.75$ | A1 | |
| | Substitute appropriate values to find $\ln A$ | M1 | |
| | Obtain $\ln A = 1.335...$ and hence $A = 3.8$ | A1 | |
| | | 5 | |

47. 9709_s19_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use logarithm subtraction property to produce logarithm of quotient | M1 | |
| | Factorise at least as far as $x(x^2-4)$ and $x(x-2)$ or use correct algebraic long division to obtain a quotient of $x+2$ and a remainder of 0 from correct working | B1 | Allow B1 either before or after application of log property<br>Allow B1 for equivalent using factorisation then use of addition rule<br>Allow B1 for $\frac{(x+2)(x^2-2x)}{(x^2-2x)}$ |
| | Obtain final answer $\ln(x+2)$ using correct process | A1 | With no errors seen |
| | | 3 | |

48. 9709_w19_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply non-modular equation $(4x+5)^2 = (x-7)^2$ or pair of different linear equations | B1 | |
| | Attempt solution of 3-term quadratic equation or pair of linear equations | M1 | |
| | Obtain $\frac{2}{5}$ and $-4$ | A1 | **SC** For $x = -4$ only, from correct work, allow B1 |
| | | 3 | |
| (ii) | Apply logarithms and use power law for $2^y = k$ where $k > 0$ from **(i)** | M1 | |
| | Obtain $-1.32$ only | A1 | AWRT |
| | | 2 | |

49. 9709_w19_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | $\ln y = \ln k + a\ln x$ | **B1** | SOI |
| | Equate gradient of line to $a$ | **M1** | |
| | Obtain $a = -1.39$ | **A1** | OE |
| | Substitute appropriate values into a correct equation to find $\ln k$ | **M1** | |
| | Obtain $\ln k = 4.266...$ and $k = 71.2$ | **A1** | **SC1** for gradient = $-1.39$ and no other relevant working |
| | **Alternative method for question 3** | | |
| | $\ln y = \ln k + a\ln x$ | **B1** | SOI |
| | $3.96 = \ln k + 0.22a$ | **M1** | For one correct equation |
| | $2.43 = \ln k + 1.32a$ | **M1** | For a second correct equation and attempt to solve to find one unknown |
| | Obtain $a = -1.39$ | **A1** | OE |
| | Obtain $\ln k = 4.266...$ and $k = 71.2$ | **A1** | **SC1** for gradient = $-1.39$ and no other relevant working |
| | **Alternative method for question 3** | | |
| | $e^{3.96} = k \times 0.22^a$ and $e^{2.43} = k \times 1.32^a$ | **B1** | |
| | Apply a correct method to obtain $a$ | **M1** | |
| | Obtain $a = -1.39$ | **A1** | OE |
| | Substitute appropriate values into a correct equation to find $k$ | **M1** | |
| | Obtain $k = 71.2$ | **A1** | AWRT |
| | | **5** | |

50. 9709_s18_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Attempt to solve quadratic equation in $e^x$ | **M1** | Either directly or using substitution $u = e^x$ |
| | Obtain $e^x = \frac{1}{3}$, $e^x = 27$ | **A1** | $e^x = \frac{1}{3}$, $e^x = 27$ may be implied if $u = e^x$ is stated |
| | Use correct process at least once for solving $e^x = c$ where $c > 0$ | **M1** | |
| | Obtain $-\ln 3$ from a correct solution | **A1** | Condone use of $x = e^x$ |
| | Obtain $3\ln 3$ from a correct solution | **A1** | |
| | | **5** | |

51. 9709_s18_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Either | | |
| | State or imply equation $\ln y = \ln A + \ln B \ln x$ | B1 | |
| | Equate gradient of line to $\ln B$ | M1 | |
| | Obtain $\ln B = 1.6486...$ and hence $B = 5.2$ | A1 | |
| | Substitute appropriate values to find $\ln A$ | M1 | |
| | Obtain $\ln A = 1.2809...$ and hence $A = 3.6$ | A1 | |
| | Or | | |
| | State or imply equation $\ln y = \ln A + \ln B \ln x$ | B1 | |
| | Use given coordinates to obtain a correct equation | B1 | Equations are $4.908 = \ln A + 2.2 \ln B$ and $11.008 = \ln A + 5.9 \ln B$ |
| | Use given coordinates to obtain a second correct equation and attempt to solve both equations simultaneously to obtain at least one of the unknowns $\ln A$ or $\ln B$ | M1 | |
| | Obtain $\ln B = 1.6486...$ and hence $B = 5.2$ | A1 | |
| | Obtain $\ln A = 1.2809...$ and hence $A = 3.6$ | A1 | |
| | Or | | |
| | Use given coordinates to obtain a correct equation | B1 | Equations are $e^{4.908} = AB^{2.2}$ and $e^{11.008} = AB^{5.9}$ |
| | Use given coordinates to obtain a second correct equation | B1 | |
| | Solve to obtain B | M1 | M mark dependent on both previous B marks |
| | $B = 5.2$ | A1 | |
| | $A = 3.6$ | A1 | |
| | | 5 | |

52. 9709_s18_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use $2\ln(2x) = \ln(4x^2)$ | B1 | |
| | Use law for addition or subtraction of logarithms | M1 | |
| | Obtain correct equation $\frac{4x^2}{x+3} = 16$ or equivalent | A1 | With no logarithms involved |
| | Solve 3-term quadratic equation | M1 | Dependent on previous M1 |
| | Conclude with $x = 6$ and, finally, no other solutions | A1 | |
| | | 5 | |
| (ii) | Apply logarithms and use power law for $2^u = k$ or $2^{u+1} = 2k$ where $k > 0$ | M1 | |
| | Obtain 2.585 | A1 | |
| | | 2 | |

53. 9709_w18_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply non-modular equation $(9x-2)^2=(3x+2)^2$ or pair of linear equations | B1 | |
| | Attempt solution of quadratic equation or of 2 linear equations | M1 | |
| | Obtain 0 and $\frac{2}{3}$ | A1 | SC: B1 for one correct solution |
| | | 3 | |
| (ii) | Apply logarithms and use power law for $3^y=k$ where $k>0$ | M1 | Must be using their answers to part **(i)** |
| | Obtain $-0.369$ | A1 | |
| | | 2 | |

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54. 9709_w18_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Recognise $9^x$ as $(3^x)^2$ or $3^{2x}$ | B1 | May be implied by $3^x(3^x+1)(=240)$ |
| | Attempt solution of quadratic equation in $3^x$ | *M1 | Perhaps using substitution $u=3^x$ |
| | Obtain, finally, $3^x=15$ only | A1 | |
| | Apply logarithms and use power law for $3^x=k$ where $k>0$ | M1 | Dependent *M, need to see $x\ln 3=\ln k$, $x=\log_3 k$ oe |
| | Obtain 2.465 | A1 | May be done using $9^{\frac{x}{2}}$, same processes |
| | | 5 | |

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55. 9709_m17_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use $2\ln(2x)=\ln(2x)^2$ | *M1 | |
| | Use addition or subtraction property of logarithms | *M1 | |
| | Obtain $4x^2=(x+3)(3x+5)$ or equivalent without logarithms | A1 | |
| | Solve 3-term quadratic equation | DM1 | dep *M *M |
| | Conclude with $x=15$ only | A1 | |
| | **Total:** | 5 | |

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56. 9709_m17_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply non-modulus inequality $(2x-5)^2 < (x+3)^2$ or corresponding equation or pair of linear equations | **B1** | |
| | Attempt solution of 3-term quadratic inequality or equation or of 2 linear equations | **M1** | |
| | Obtain critical values $\frac{2}{3}$ and 8 | **A1** | |
| | State answer $\frac{2}{3} < x < 8$ | **A1** | |
| | **Total:** | **4** | |
| (ii) | Attempt to find $y$ from $\ln y$ = upper limit of answer to part **(i)** | **M1** | |
| | Obtain 2980 | **A1** | |
| | **Total:** | **2** | |

57. 9709_s17_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Take logarithms of both sides and apply power law to both sides | **M1** | Allow $y = \frac{\log 5}{4\log 3}$ for **M1 A1** |
| | Rearrange to the form $y = \frac{\ln 5}{4\ln 3}x$ or equivalent | **A1** | |
| | Obtain $m = 0.366$ | **A1** | |
| | **Total:** | **3** | |

58. 9709_s17_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Apply logarithms to both sides and apply power law | ***M1** | |
| | Obtain $(x+4)\log 3 = 2x\log 5$ or equivalent | **A1** | |
| | Solve linear equation for $x$ | **DM1** | dep *M |
| | Obtain 2.07 | **A1** | Allow greater accuracy |
| | **Total:** | **4** | |

59. 9709_s17_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | State or imply $\ln y = \ln K - 2x\ln a$ | B1 | |
| | *EITHER:* | | |
| | Obtain $-0.525$ as gradient of line | (M1 | |
| | Equate their $-2\ln a$ to their gradient and solve for $a$ | M1 | Allow $2\ln a$ = their gradient for **M1** |
| | Obtain $a = 1.3$ | A1 | |
| | Substitute to find value of $K$ | M1 | |
| | Obtain $K = 8.4$ | A1) | |
| | *OR:* | | |
| | Obtain two equations using coordinates correctly | (M1 | |
| | Solve these equations to obtain $2\ln a$ or equivalent | M1 | |
| | Obtain $a = 1.3$ | A1 | |
| | Substitute to find value of $K$ | M1 | |
| | Obtain $K = 8.4$ | A1) | |
| | **Total:** | 6 | |

60. 9709_w17_MS_21 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use subtraction or addition property of logarithms | *M1 | |
| | Obtain $\frac{3x+1}{x+2} = e$ or equivalent with no presence of logarithm | A1 | |
| | Use correct process to solve equation | DM1 | |
| | Obtain $\frac{2e-1}{3-e}$ or exact equivalent | A1 | |
| | | 4 | |

61. 9709_w17_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Take logarithms of both sides and apply power law | **M1** | Condone incorrect inequality signs until final answer. The first 6 marks are for obtaining the correct critical values. |
| | Obtain $2x < \frac{\ln 80}{\ln 1.3}$ or equivalent using $\log_{10}$ | **A1** | |
| | Obtain $x = 8.35...$ | **A1** | |
| | State or imply non-modulus inequality $(3x-1)^2 > (3x-10)^2$ or corresponding equation or linear equation $3x-1 = -(3x-10)$ | **B1** | |
| | Attempt solution of inequality or equation (obtaining 3 terms when squaring each bracket or solving linear equation with signs of $3x$ different) | **M1** | |
| | Obtain $x = \frac{11}{6}$ or $x = 1.83...$ | **A1** | |
| | Conclude $1.83 < x < 8.35$ | **A1** | |
| | | **7** | |

62. 9709_w17_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Introduce logarithms to both sides and use power law | ***M1** | |
| | Obtain $(3x-1)\log 5 = 4x\log 2$ or equivalent | **A1** | Allow **A1** for poor use of brackets if recovered later |
| | Solve linear equation for $x$ | **DM1** | dep *M |
| | Obtain 0.783 | **A1** | Allow 3 sf or better |
| | | **4** | |

63. 9709_m16_MS_22 Q: 3

Use $2\ln x = \ln x^2$ **B1**
Use law for addition or subtraction of logarithms **M1**
Obtain $x^2 = (3+x)(2-x)$ or equivalent with no logarithms **A1**
Solve 3-term quadratic equation **M1**
Obtain $x = \frac{3}{2}$ and no other solutions **A1** [5]

64. 9709_s16_MS_21 Q: 3

| | | |
|---|---|---|
| Rearrange to $3e^{2x} - 14e^{x} + 8 = 0$ or equivalent involving substitution | **B1** | |
| Solve quadratic equation in $e^x$ to find two values of $e^x$ | ***M1** | |
| Obtain $\frac{2}{3}$ and 4 | **A1** | |
| Use natural logarithms to solve equation of form $e^x = k$ where $k > 0$ dep on | **DM1** | |
| Allow M mark if left in exact form | **M1** | |
| Obtain $-0.405$ | **A1** | |
| Obtain 1.39 | **A1** | [6] |

65. 9709_s16_MS_22 Q: 1

| | | |
|---|---|---|
| Use power law for logarithms correctly at least once | **M1** | |
| Obtain $3x\log 5 = 4y\log 7$ or $3x\ln 5 = 4y\ln 7$ or equivalent | **A1** | |
| Obtain 1.612 | **A1** | [3] |

66. 9709_w16_MS_21 Q: 1

| | | | |
|---|---|---|---|
| **(i)** | Carry out method for solving quadratic equation in $3^x$<br>Obtain at least $3^x = 7$<br>Use logarithms to solve an equation of the form $3^x = k$ where $k > 0$<br>Obtain 1.77 | **M1**<br>**A1**<br>**M1**<br>**A1** | [4] |
| **(ii)** | State $\pm 1.77$, following positive answer from part (i) | **B1**√ | [1] |

67. 9709_w16_MS_21 Q: 2

| | | | |
|---|---|---|---|
| | State or imply $\ln y = \ln A + px$<br>Equate gradient of line to $p$<br>Obtain $p = 0.32$<br>Substitute to find $A$<br>Obtain $A = 4.81$<br><br>OR 1:<br>$3.17 = \ln A + 5p$ or $4.77 = \ln A + 10p$<br>Correct attempt to obtain $\ln A$ or $p$<br>Correct attempt to obtain the other unknown<br>Obtain $A = 4.81$<br>Obtain $p = 0.32$<br><br>OR 2:<br>$e^{3.17} = Ae^{5p}$ or $e^{4.77} = Ae^{10p}$<br>Correct attempt to obtain p<br>Correct attempt to get A<br>Obtain $A = 4.81$<br>Obtain $p = 0.32$ | **B1**<br>**M1**<br>**A1**<br>**M1**<br>**A1**<br><br><br>**B1**<br>**M1**<br>**M1**<br>**A1**<br>**A1**<br><br><br>**B1**<br>**M1**<br>**M1**<br>**A1**<br>**A1** | [5] |

68. 9709_w16_MS_22 Q: 2

| | | | | |
|---|---|---|---|---|
| **(i)** | Use $4^y = 2^{2y}$ | **B1** | | |
| | Attempt solution of quadratic equation in $2^y$ | **M1** | | |
| | Obtain finally $2^y = 7$ only | **A1** | [3] | |
| **(ii)** | Apply logarithms to solve equation of form $2^y = k$ where $k > 0$ | **M1** | | Must be using their positive answer for (i) |
| | Obtain 2.81 | **A1** | [2] | |

69. 9709_w16_MS_23 Q: 2

| | | | |
|---|---|---|---|
| | State or imply $\ln y = \ln K + p \ln x$ | **B1** | |
| | Calculate gradient of line | **M1** | |
| | Obtain $p = 1.35$ | **A1** | |
| | Substitute to find $K$ | **M1** | |
| | Obtain $K = 7.11$ or $K = 7.12$ | **A1** | [5] |

70. 9709_w16_MS_23 Q: 6

| | | | |
|---|---|---|---|
| **(i)** | State $\frac{dx}{dt} = \frac{1}{t+1}$ | **B1** | |
| | Use product rule for derivative of $y$ | **M1** | |
| | Obtain $2t\ln t + t$ or equivalent | **A1** | |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** | |
| | Obtain $(t+1)(2t\ln t + t)$ | **A1** | [5] |
| **(ii)** | Solve $2\ln t + 1 = 0$ | **M1** | |
| | Obtain $t = e^{-\frac{1}{2}}$ | **A1** | [2] |
| **(iii)** | Identify $t = 1$ only | **B1** | |
| | Obtain 2 | **B1** | [2] |

71. 9709_s15_MS_21 Q: 1

**(i)** State or imply equation $(3x+4)^2 = (3x-11)^2$ or $3x + 4 = -(3x-11)$ B1
Attempt solution of 'quadratic' equation or linear equation M1
Obtain $x = \frac{7}{6}$ or equivalent (and no other solutions) A1 [3]

**(ii)** Use logarithms to solve equation of form $2^y$ = their answer to (i) ( must be + ve) M1
Obtain 0.222 (and no other solutions) A1 [2]

72. 9709_s15_MS_21 Q: 2

| | | |
|---|---|---|
| State or imply that $\ln y = \ln A + p(x-1)$ | B1 | |
| Equate gradient to $p$ or obtain two equations for $\ln A$ and $p$ | M1 | |
| Obtain $p = 0.44$ | A1 | |
| Substitute values correctly, to find value of $\ln A$ | DM1 | |
| Obtain $A = 3.2$ | A1 | [5] |
| Alternative: | | |
| Obtain an equation either $e^{1.6} = Ae^{p}$ or $e^{2.92} = Ae^{4p}$ | M1 | |
| Obtain both equations correctly | A1 | |
| Solve to obtain $p = 0.44$ | A1 | |
| Substitute value correctly to find $A$ | DM1 | |
| Obtain $A = 3.2$ | A1 | [5] |

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73. 9709_s15_MS_22 Q: 1

| | | | |
|---|---|---|---|
| **(i)** | Introduce logarithms and use power law | M1 | |
| | Obtain $x = 21.6$ | A1 | [2] |
| **(ii)** | Obtain or imply –21.6 or –21 as lower value | B1 | |
| | State 43 | B1 | [2] |

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74. 9709_s15_MS_22 Q: 7

| | | | |
|---|---|---|---|
| **(a)** | Differentiate $4\ln y$ to obtain $\frac{4}{y} \times \frac{dy}{dx}$ | B1 | |
| | Differentiate $6xy$ to obtain $6y + 6x\frac{dy}{dx}$ | B1 | |
| | Substitute 1 and 1 and solve for $\frac{dy}{dx}$ | M1 | |
| | Obtain $-\frac{9}{10}$ or equivalent | A1 | [4] |
| **(b)** | Obtain $\frac{dx}{dt} = -10t^{-2} - 1$ | B1 | |
| | Obtain derivative of form $k(2t-1)^{-\frac{1}{2}}$ for $\frac{dy}{dt}$ | M1 | |
| | Obtain correct $(2t-1)^{-\frac{1}{2}}$ | A1 | |
| | Identify value of $t$ as 5 | B1 | |
| | Obtain expression for $\frac{dy}{dx}$ correctly,with numerical value of $t$ substituted | M1 | |
| | Obtain $-\frac{5}{21}$ or exact equivalent | A1 | [6] |

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75. 9709_w15_MS_21 Q: 1

| | | |
|---|---|---|
| Introduce logarithms and use power law twice | **M1*** | |
| Obtain $(x+3)\log 5 = (x-1)\log 7$ or equivalent | **A1** | |
| Solve linear equation for $x$ | **M1 dep** | |
| Obtain 20.1 | **A1** | [4] |

---

76. 9709_w15_MS_22 Q: 1

| | | | | |
|---|---|---|---|---|
| **(i)** | Either | Square both sides to obtain three-term quadratic equation | **M1** | |
| | | Solve three-term quadratic equation to obtain two values | **M1** | |
| | | Obtain –1 and $\frac{7}{3}$ | **A1** | |
| | Or | Obtain $\frac{7}{3}$ from graphical method, inspection or linear equation | **B1** | |
| | | Obtain –1 similarly | **B2** | [3] |
| **(ii)** | | Use logarithmic method to solve an equation of the form $5^y = k$ where k > 0 | **M1** | |
| | | Obtain 0.526 and no others | **A1** | [2] |

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77. 9709_w15_MS_22 Q: 3

| | | |
|---|---|---|
| State or imply that $\ln y = \ln K + m \ln x$ | **B1** | |
| Form a numerical expression for gradient of line | **M1** | |
| Obtain –1.39 or –1.4 | **A1** | |
| Use their gradient value and one point correctly to obtain intercept | **M1** | |
| Obtain value for $\ln K$ between 4.26 and 4.28 | **A1** | |
| Obtain $K = 71$ or $K = 72$ or value rounding to either with no error noted | **A1** | [6] |

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78. 9709_w15_MS_23 Q: 2

| | | | | |
|---|---|---|---|---|
| **(i)** | Either | State or imply non-modulus equation $(2x+3)^2 = (x+8)^2$ or corresponding pair of linear equations | **B1** | |
| | | Solve 3-term quadratic equation or 2 linear equations | **M1** | |
| | | Obtain $x = -\frac{11}{3}$ and $x = 5$ | **A1** | |
| | Or | Obtain $x = 5$ from graphical method, inspection, equation, … | **B1** | |
| | | Obtain $x = -\frac{11}{3}$ similarly | **B2** | [3] |
| **(ii)** | | Use logarithms to solve equation of form $2^y = k$ where $k > 0$ | **M1** | |
| | | Obtain 2.32 | **A1** | [2] |

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79. 9709_s20_MS_21 Q: 5

| | | |
|---|---|---|
| (a) | Differentiate using the product rule to obtain $ax^2 \cos 2x - bx^3 \sin 2x$ | M1 |
| | Obtain $3x^2 \cos 2x - 2x^3 \sin 2x$ | A1 |
| | Equate first derivative to zero and confirm $x = \sqrt[3]{1.5x^2 \cot 2x}$ AG | A1 |
| | | 3 |
| (b) | Consider sign of $x - \sqrt[3]{1.5x^2 \cot 2x}$ or equivalent for 0.59 and 0.60 | M1 |
| | Obtain –0.009... and 0.005... or equivalents and justify conclusion | A1 |
| | | 2 |
| (c) | Use iteration correctly at least once | M1 |
| | Obtain final answer 0.596 | A1 |
| | Show sufficient iterations to 5 sf to justify answer or show sign change in interval [0.5955, 0.5965] | A1 |
| | | 3 |

80. 9709_s20_MS_22 Q: 6

| | | |
|---|---|---|
| (a) | Substitute $x=-3$, equate to zero and attempt solution for $a$ | M1 |
| | Obtain $a=17$ | A1 |
| | | 2 |
| (b) | Divide by $x+3$ at least as far as the $x$ term | M1 |
| | Obtain $6x^2-x-1$ | A1 |
| | Conclude $(x+3)(3x+1)(2x-1)$ | A1 |
| | | 3 |
| (c) | Attempt solution of $\sin\theta=k$ where $-1\leqslant k\leqslant 1$ | M1 |
| | Obtain 199.5 | A1 |
| | Obtain 340.5 | A1 |
| | | 3 |

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81. 9709_w20_MS_21 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Use $\sin 2\theta = 2\sin\theta\cos\theta$ | B1 | |
| | Obtain $\sin\theta=\frac{1}{6}$ | B1 | |
| | | 2 | |
| (b) | Use correct identity or identities to find value of $\sec\theta$ | M1 | |
| | Obtain $\frac{6}{\sqrt{35}}$ or exact equivalent | A1 | |
| | | 2 | |
| (c) | Use correct identity or identities to find value of $\cos 2\theta$ | M1 | |
| | Obtain $\frac{17}{18}$ or exact equivalent | A1 | |
| | | 2 | |

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82. 9709_w20_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use $\cot\theta = \frac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ | B1 | SOI |
| | Simplify to obtain $\cos\theta = k$ where $0 < k < 1$ | M1 | |
| | Obtain $\cos\theta = \frac{3}{7}$ and hence $\theta = 64.6$ and no other solutions in the range | A1 | |
| | **Alternative method for question 1** | | |
| | Use identity $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ | B1 | |
| | Simplify to obtain $\tan\theta = k_1$ or $\sin\theta = k_2$ where $0 < k_2 < 1$ | M1 | |
| | Obtain $\tan\theta = \frac{1}{3}\sqrt{40}$ or $\sin\theta = \frac{1}{7}\sqrt{40}$ and hence $\theta = 64.6$ and no other solutions in the range | A1 | |
| | | 3 | |

83. 9709_m19_MS_22 Q: 1

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use identity $\sec^2\theta = 1 + \tan^2\theta$ | B1 | |
| | Attempt solution of quadratic equation to find two values of $\tan\theta$ | M1 | |
| | Obtain $\tan\theta = -\frac{1}{2},\ 3$ | A1 | |
| | Obtain 71.6 and 153.4 and no others between 0 and 180 | A1 | |
| | | 4 | |

84. 9709_s19_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply $\operatorname{cosec} 2\theta = \dfrac{1}{2\sin\theta\cos\theta}$ | B1 | |
| | Attempt to express left-hand side in terms of $\sin\theta$ and $\cos\theta$ only | M1 | |
| | Simplify to confirm $\operatorname{cosec}^2\theta$ **AG** | A1 | |
| | | 3 | |
| (ii) | Use identity to express left-hand side in terms of $\sin 30$ or $\operatorname{cosec} 30$ | M1 | |
| | Obtain $\dfrac{2}{\sin 30}$ or $2\operatorname{cosec} 30$ and confirm 4 **AG** | A1 | |
| | | 2 | |
| (iii) | Solve quadratic equation of the form $k\operatorname{cosec}^2\dfrac{\phi}{2} + \operatorname{cosec}\dfrac{\phi}{2} - 12 = 0$ or | *M1 | Allow sign errors |
| | $12\sin^2\dfrac{\phi}{2} - \sin\dfrac{\phi}{2} - k = 0$ correctly for $\operatorname{cosec}\frac{1}{2}\phi$ or $\sin\frac{1}{2}\phi$ to find two values of $\sin\frac{1}{2}\phi$ or $\operatorname{cosec}\frac{1}{2}\phi$<br>Obtain $\sin\frac{1}{2}\phi = -\frac{1}{4},\ \frac{1}{3}$ | A1 | |
| | Use correct process to find at least one correct value of $\phi$ from $\sin\frac{1}{2}\phi = \pm\frac{1}{4},\ \pm\frac{1}{3}$ | DM1 | Allow for any rounded or truncated value |
| | Obtain any two of $-331.0, -29.0, 38.9, 321.1$ | A1 | Allow greater accuracy |
| | Obtain all four values and no others between $-360$ and $360$ | A1 | Allow greater accuracy |
| | | 5 | |

85. 9709_s19_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a)(i) | State $R = \sqrt{32}$ or equivalent or 5.657… | B1 | |
| | Use appropriate trigonometry to find $\alpha$ | M1 | |
| | Obtain $\alpha = 45$ | A1 | |
| | | 3 | |
| (a)(ii) | Carry out correct process to find one value of $\theta$ | M1 | |
| | Obtain 17.1 | A1 | Ignore other positive values greater than 17.1 |
| | | 2 | |
| (b) | Use or imply $\cot 2x = \dfrac{1}{\tan 2x}$ | B1 | |
| | Use identity of form $\tan 2x = \dfrac{\pm 2\tan x}{1 \pm \tan^2 x}$ to obtain equation in $\tan x$ | M1 | |
| | Obtain $6\tan^2 x + 10\tan x - 4 = 0$ or equivalent | A1 | |
| | Attempt solution of 3-term quadratic equation for $\tan x$ | M1 | |
| | Obtain $\tan x = \frac{1}{3}$ and hence 0.32 | A1 | Allow greater accuracy |
| | Obtain $\tan x = -2$ and hence 2.03 and no others between 0 and $\pi$ | A1 | Allow greater accuracy |
| | | 6 | |

86. 9709_w19_MS_21 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Express equation as $\frac{1}{\cos\alpha\sin\alpha}=7$ | B1 | OE; May be implied by subsequent work |
| | Attempt use of identity for $\sin 2\alpha$ or attempt to obtain a quadratic equation in terms of any one of the following: $\sin^2\alpha$, $\cos^2\alpha$, $\cot^2\alpha$ or $\tan^2\alpha$ | M1 | From equation of form $\sin 2\alpha = k$ where $0<k<1$ or from use of correct identities |
| | Obtain $\sin 2\alpha = \frac{2}{7}$ or a correct 3 term quadratic equation, equated to zero in any one of the following: $\sin^2\alpha$, $\cos^2\alpha$, $\cot^2\alpha$ or $\tan^2\alpha$ | A1 | |
| | Attempt correct process to find at least one correct value of $\alpha$ | M1 | |
| | Obtain 8.3 and 81.7 and no others between 0 and 90 | A1 | |
| | | 5 | |
| (b) | Simplify left-hand side to obtain $2\sin\beta\cos 20°$ | B1 | |
| | Attempt to form equation where $\tan\beta$ is only variable, $\tan\beta \neq 3$ | M1 | |
| | Obtain $\tan\beta = \frac{3}{\cos 20°}$ | A1 | OE |
| | Obtain $\beta = 72.6$ and no others between 0 and 90 | A1 | |
| | | 5 | |

87. 9709_w19_MS_22 Q: 8

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State $R = 1.3$ or $\frac{10}{3}$ | B1 | Not $\sqrt{1.69}$ |
| | Use appropriate trigonometry to find $\alpha$ | M1 | AWRT $\pm 1.18$ rads, AWRT $\pm 0.391$ rads, AWRT $\pm 67.4°$, AWRT $\pm 22.6°$ |
| | Obtain 67.38 with no errors seen | A1 | AWRT |
| | | 3 | |
| (ii) | Carry out correct method to find one value of $\theta$ between 0 and 360 | M1 | |
| | Obtain 240.6 (or 344.6) | A1 | |
| | Carry out correct method to find second value of $\theta$ between 0 and 360 | M1 | Must be using either degrees throughout or radians throughout for M marks |
| | Obtain 344.6 (or 240.6) | A1 | |
| | | 4 | |
| (iii) | Recognise expression as $[3-2R\cos(\theta+\alpha)]^2$ | M1 | |
| | Obtain $[3-2\times(-1.3)]^2$ and hence 31.36 or 31.4 | A1 | |
| | Obtain $[3-2\times 1.3]^2$ and hence 0.16 | A1 | |
| | | 3 | |

88. 9709_w18_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | State $\frac{1}{\cos^2\theta}=\frac{3}{\sin\theta}$ or $1+\tan^2\theta=\frac{3}{\sin\theta}$ | B1 | |
| | Produce quadratic equation in $\sin\theta$ | M1 | Dependent on B1 |
| | Solve 3-term quadratic equation to find value between –1 and 1 for $\sin\theta$ | M1 | Dependent on first M1 |
| | Obtain $\sin\theta=\frac{1}{6}(-1+\sqrt{37})$ and hence 57.9 | A1 | |
| | Obtain 122.1 and no others between 0 and 180 | A1 | |
| | | 5 | |

89. 9709_w18_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute $-\frac{3}{2}$ and simplify | M1 | Allow use of identity assuming a factor of $2x+3$ to obtain a quadratic factor. Need to see use of 4 equations to verify quadratic for M1, A1 for conclusion. Allow verification by expansion.<br>Allow use of identity including a remainder to obtain a quadratic factor and a remainder of zero. Need to see use of 4 equations for M1, A1 for conclusion. Allow verification by expansion.<br>Allow use of long division , must reach a remainder of zero for M1 |
| | Obtain $-27+9+15+3$ or equivalent, hence zero and conclude , may have explanation at start of working | A1 | Need powers of $-\frac{3}{2}$ evaluating for A1<br>AG; necessary detail needed |
| | | 2 | |
| (ii) | Use $\cos 2\theta = 2\cos^2\theta - 1$ | B1 | |
| | Simplify $a\cos^2\theta + b = \frac{6\cos\theta - 5}{2\cos\theta + 1}$ to polynomial form | M1 | |
| | Obtain $8\cos^3\theta + 4\cos^2\theta - 10\cos\theta + 3 = 0$ | A1 | AG; necessary detail needed, must be completely correct with no poor use of brackets for A1 |
| | | 3 | |
| (iii) | Attempt either division by $2x+3$ and reach partial quotient $x^2+kx$ or use of identity or inspection | *M1 | Or equivalent using $\cos\theta$ or $c$ |
| | Obtain quotient $4x^2-4x+1$ | A1 | Or equivalent |
| | Obtain factorised form $(2x+3)(2x-1)^2$ | A1 | Or equivalent, may be implied by later work |
| | Solve for $\cos\theta = k$ to find at least one value between 0 and 360 | M1 | Dependent *M |
| | Obtain 60 and 300 and no others | A1 | SC1: Equation solver used to obtain 60 and 300 and no others, then 5/5<br>SC2:Equation solver used to obtain 60 then 4/5<br>SC 3: $\cos\theta=0.5$, $(\cos\theta=-1.5)$ seen implies first 3 marks. |
| | | 5 | |

90. 9709_m17_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use identity $\cot\theta = \dfrac{1}{\tan\theta}$ | B1 | |
| | Attempt use of identity for $\tan 2\theta$ | M1 | |
| | Confirm given $\tan^2\theta = \frac{3}{4}$ | A1 | |
| | **Total:** | 3 | |
| (ii) | Obtain 40.9 | B1 | |
| | Obtain 139.1 | B1 | |
| | **Total:** | 2 | |

91. 9709_s17_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State $R = 3$ | B1 | Allow marks for (i) if seen in (ii) |
| | Use appropriate trigonometric formula to find $\alpha$ | M1 | |
| | Obtain 48.19 with no errors seen | A1 | |
| | **Total:** | 3 | |
| (ii) | Carry out evaluation of $\cos^{-1}\frac{1}{3}(=70.528...)$ | M1 | M1 for $\cos^{-1}\left(\dfrac{1}{R}\right)$ |
| | Obtain correct answer 118.7 | A1 | |
| | Carry out correct method to find second answer | M1 | |
| | Obtain 337.7 and no others between 0 and 360 | A1 | |
| | **Total:** | 4 | |

92. 9709_w17_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use $\cos 2\theta = 2\cos^2\theta - 1$ | B1 | |
| | Obtain $10\cos^3\theta = 4$ or equivalent | B1 | |
| | Use correct process to find at least one value of $\theta$ from equation of form $k_1\cos^3\theta = k_2$ | M1 | |
| | Obtain 42.5 | A1 | |
| | Obtain 317.5 and no others between 0 and 360 | A1 | |
| | | 5 | |

93. 9709_s16_MS_21 Q: 2

| | | |
|---|---|---|
| Use $\cot\theta = 1 \div \tan\theta$ | B1 | |
| Form equation involving $\tan\theta$ only and with no denominators involving $\theta$ | M1 | |
| Obtain $\tan^2\theta = \frac{2}{7}$ | A1 | |
| Obtain 28.1 | A1 | |
| Obtain 151.9 | A1 | [5] |
| Allow other valid methods | | |

94. 9709_s16_MS_22 Q: 4

| | | | | |
|---|---|---|---|---|
| **(i)** | | State $\sin\theta\cos 60 + \cos\theta\sin 60 + \sin\theta\cos 120 + \cos\theta\sin 120$ | *B1 | |
| | | Use $\sin 60 = \sin 120 = \frac{1}{2}\sqrt{3}$ and $\cos 60 = \frac{1}{2}$, $\cos 120 = -\frac{1}{2}$ | *B1 | |
| | | Confirm result $\sqrt{3}\cos\theta$, dependent on *B *B | DB1 | [3] |
| **(ii)** | **(a)** | $\cos 45$ seen | *B1 | |
| | | State $\sqrt{\frac{3}{2}}$ or $\frac{1}{2}\sqrt{6}$ or exact equivalent, dependent *B | DB1 | [2] |
| | **(b)** | Carry out correct process to find at least one value of $\theta$ from $\cos^2\theta = k$ | M1 | |
| | | Obtain 40.6 | A1 | |
| | | Obtain 139.4 | A1 | [3] |

95. 9709_w16_MS_21 Q: 7

| | | | |
|---|---|---|---|
| **(i)** | Substitute $x = -3$, equate to zero and obtain $27a + 3b = 39$ or equivalent<br>Substitute $x = -2$ and equate to 18<br>Obtain $8a + 2b = 6$ or equivalent<br>Solve a relevant pair of linear equations for $a$ and $b$<br>Obtain $a = 2$ and $b = -5$ | B1<br>M1<br>A1<br>M1<br>A1 | [5] |
| **(ii) (a)** | Attempt division by $x + 3$ at least as far as $2x^2 + kx$<br>Obtain quotient $2x^2 - 3x + 4$<br>Calculate discriminant of 3-term quadratic expression, or equivalent<br>Obtain $-23$ and conclude appropriately | M1<br>A1<br>M1<br>A1 | [4] |
| **(b)** | State $\cos y = -\frac{1}{3}$<br>Obtain 109.5, dependent *B<br>Obtain $-109.5$ and no others between $-180$ and 180, dependent *B | *B1<br>B1<br>DB1 | [3] |

96. 9709_w16_MS_22 Q: 7

| | | | | |
|---|---|---|---|---|
| (i) | Use correct addition formula for either $\cos(\theta+\frac{1}{6}\pi)$ or, after diffn, $\sin(\theta+\frac{1}{6}\pi)$ | B1 | | Condone 'missing brackets' |
| | Differentiate to obtain $\frac{dy}{d\theta}$ of form $k_1\sin\theta+k_2\cos\theta$ or $k\sin(\theta+\frac{1}{6}\pi)$ | M1 | | |
| | Divide attempt at $\frac{dy}{d\theta}$ by attempt at $\frac{dx}{d\theta}$ | M1 | | |
| | Obtain $\dfrac{-\frac{3\sqrt{3}}{2}\sin\theta-\frac{3}{2}\cos\theta}{4\cos\theta}$ or equivalent | A1 | | |
| | Simplify to obtain $-\frac{3}{8}(1+\sqrt{3}\tan\theta)$ | A1 | [5] | |
| (ii) | Identify $\theta=0$ | B1 | | soi |
| | Substitute 0 into formula for $\frac{dy}{dx}$ and take negative reciprocal | M1 | | be implied by $y=1+\dfrac{3\sqrt{3}}{2}$ or 3.6 |
| | Obtain gradient of normal $\frac{8}{3}$ | A1 | | Must be from correct **(i)** |
| | Form equation of normal through point $(0,1+\frac{3\sqrt{3}}{2})$ | M1 | | |
| | Obtain $y=\frac{8}{3}x+1+\frac{3\sqrt{3}}{2}$ or equivalent | A1 | [5] | |

97. 9709_w16_MS_23 Q: 7

| | | | |
|---|---|---|---|
| (i) | State $\dfrac{3}{\cos\theta}+\dfrac{4}{\sin\theta}$ | B1 | |
| | Use identity for $\sin 2\theta$ and obtain expression of form $a\sin\theta+b\cos\theta$ | M1 | |
| | Obtain $6\sin\theta+8\cos\theta$ | A1 | [3] |
| (ii) | State $R=10$, following their $a\sin\theta+b\cos\theta$ | B1√ | |
| | Use appropriate trigonometry to find $\alpha$ | M1 | |
| | Obtain 53.1(3) with no errors seen | A1 | [3] |
| (iii) | Carry out correct process to find one angle between 0 and 360 | M1 | |
| | Obtain 82.4 or 82.5 | A1 | |
| | Carry out correct process to find second angle between 0 and 360 | M1 | |
| | Obtain 351.3 and no others between 0 and 360 | A1 | [4] |

98. 9709_s15_MS_22 Q: 3

| | | | |
|---|---|---|---|
| **(i)** | Use identity $\sec^2\theta = 1+\tan^2\theta$ | B1 | |
| | Solve three-term quadratic equation in $\tan\theta$ | M1 | |
| | Obtain at least $\tan\theta = \frac{5}{2}$ | A1 | [3] |
| **(ii)** | Substitute numerical values into $\tan(A+B)$ identity | M1 | |
| | Obtain $\frac{\frac{5}{2}+(-1)}{1-\frac{5}{2}(-1)}$ or equivalent, following their positive answer from part (i) | A1√ | |
| | Obtain $\frac{3}{7}$ or exact equivalent and no other answers | A1 | [3] |

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99. 9709_w15_MS_21 Q: 3

| | | | |
|---|---|---|---|
| **(i)** | State or imply $R = 17$ | **B1** | |
| | Use appropriate formula to find $\alpha$ | **M1** | |
| | Obtain 61.93 | **A1** | [3] |
| **(ii)** | Attempt to find at least one value of $\theta + \alpha$ | **M1** | |
| | Obtain one correct value of $\theta$ (97.4 or 318.7) | **A1** | |
| | Carry out correct method to find second answer | **M1** | |
| | Obtain second correct value and no others between 0 and 360 | **A1** | [4] |

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100. 9709_w15_MS_22 Q: 4

| | | | |
|---|---|---|---|
| **(i)** | Substitute $x = -2$ and equate to zero | **M1** | |
| | Solve equation to confirm $a = -4$ | **A1** | [2] |
| **(ii) (a)** | Find quadratic factor by division, inspection, identity, … | **M1** | |
| | Obtain $6x^2 - x - 2$ | **A1** | |
| | Conclude $(x+2)(3x-2)(2x+1)$ | **A1** | [3] |
| **(b)** | State or imply at least $\sec\theta = -2$ and attempt solution | **M1** | |
| | Obtain 120° and no others in range | **A1** | [2] |

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101. 9709_w15_MS_23 Q: 6

| | | | |
|---|---|---|---|
| **(i)** | State or imply $R = 3$ | **B1** | |
| | Use appropriate formula to find $\alpha$ | **M1** | |
| | Obtain 41.81° | **A1** | [3] |
| **(ii) (a)** | Attempt to find one correct value of $\theta + \alpha$ | **M1** | |
| | Obtain one correct value (30.7 or 245.6) of $\theta$ | **A1** | |
| | Carry out correct method to find second answer | **M1** | |
| | Obtain second correct answer and no others in range | **A1** | [4] |
| **(b)** | State greatest value is 13, following their value of $R$ | **B1** | |
| | State least value is 7, following their value of $R$ | **B1** | [2] |

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102. 9709_s20_MS_21 Q: 3

| | | |
|---|---|---|
| | State $\frac{dx}{dt} = e^t + 2e^{-t}$, $\frac{dy}{dt} = 6e^{2t}$ | B1 |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ either in terms of $t$ or after substitution of $t = 0$ | *M1 |
| | Obtain gradient of tangent is 2 | A1 |
| | Attempt equation of tangent with numerical gradient and coordinates | DM1 |
| | Obtain $y = 2x + 6$ or equivalent | A1 |
| | | 5 |

103. 9709_s20_MS_22 Q: 2

| | | |
|---|---|---|
| | Differentiate using product rule to obtain $ae^{\frac{1}{2}x} + bxe^{\frac{1}{2}x}$ | *M1 |
| | Obtain correct $5e^{\frac{1}{2}x} + \frac{5}{2}xe^{\frac{1}{2}x}$ OE | A1 |
| | Equate first derivative to zero and solve for $x$ | DM1 |
| | Obtain $x$-coordinate $-2$ | A1 |
| | Obtain $y$-coordinate $-10e^{-1}$ | A1 |
| | | 5 |

104. 9709_s20_MS_22 Q: 3

| | | |
|---|---|---|
| | Differentiate $\cos 3x$ to obtain $-3\sin 3x$ | B1 |
| | Differentiate $5\sin y$ to obtain $5\cos y \frac{dy}{dx}$ | B1 |
| | Obtain $-3\sin 3x + 5\cos y \frac{dy}{dx} = 0$ OE | B1 |
| | Substitute $x$ and $y$ values to find value of first derivative | M1 |
| | Obtain $\frac{3}{5}$ | A1 |
| | | 5 |

105. 9709_w20_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Obtain $\frac{dx}{dt} = 3 - 2\cos t$ and $\frac{dy}{dt} = 5 - 4\sin t$ | B1 | |
| | Equate expression for $\frac{dy}{dx}$ to $\frac{5}{2}$ | M1 | |
| | Obtain $10\cos t - 8\sin t = 5$ | A1 | AG – sufficient working to be shown. |
| | | 3 | |
| (b) | State $R = \sqrt{164}$ or exact equivalent | B1 | |
| | Use appropriate trigonometry to find $\alpha$ | M1 | |
| | Obtain 0.675 with no errors seen | A1 | AWRT |
| | | 3 | |
| (c) | Carry out correct method to find one value of $t$ | M1 | Must be using the result from **(b)** |
| | Obtain 0.495 | A1 | AWRT |
| | Carry out correct method to find second value of $t$ | M1 | |
| | Obtain 4.44 | A1 | AWRT |
| | | 4 | |

106. 9709_w20_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Use product rule to differentiate $2e^{2x}y$ | M1 | Must be in the form $k_1ye^{2x} + k_2e^{2x}\frac{dy}{dx}$ |
| | Obtain $4e^{2x}y + 2e^{2x}\frac{dy}{dx}$ | A1 | |
| | Differentiate $-y^3$ to obtain $-3y^2\frac{dy}{dx}$ | B1 | |
| | Obtain $\frac{dy}{dx} = \frac{4e^{2x}y}{3y^2 - 2e^{2x}}$ | A1 | AG |
| | | 4 | |
| (b) | Substitute 0 and 2 to find gradient of tangent | M1 | |
| | Attempt to find equation of tangent through $(0, 2)$ with numerical gradient | M1 | |
| | Obtain $4x - 5y + 10 = 0$ or equivalent of required form | A1 | |
| | | 3 | |
| (c) | Equate numerator of derivative to zero and state that at least one of $e^{2x}$ and $y$ cannot be zero | M1 | |
| | Complete argument | A1 | |
| | | 2 | |

107. 9709_m19_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $\frac{dx}{dt} = 2 - 2\cos 2t$ | B1 | |
| | Obtain $\frac{dy}{dt} = 5 - 2\sin 2t$ | B1 | |
| | Equate attempt at $\frac{dy}{dx}$ to 2 and rearrange | M1 | |
| | Confirm equation $2\sin 2t - 4\cos 2t = 1$ | A1 | Answer given; necessary detail needed |
| | | 4 | |
| (ii) | State $R = \sqrt{20}$ or 4.47 | B1 | |
| | Use appropriate trigonometry to find $\alpha$ | M1 | |
| | Obtain $\alpha = 1.107$ with no errors seen | A1 | |
| | Carry out correct method to find value of $t$ | M1 | |
| | Obtain $t = 0.666$ | A1 | |
| | Substitute value of $t$ between 0 and $\frac{1}{2}\pi$ into expressions for $x$ and $y$ | M1 | |
| | Obtain $x = 0.361$, $y = 3.57$ | A1 | |
| | | 7 | |

108. 9709_s19_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use product rule to differentiate $x^2 \ln y$ | M1 | Allow M1 for $2x\ln y + x^2y^{-1}$ oe |
| | Obtain $2x\ln y + x^2 \times \frac{1}{y} \times \frac{dy}{dx}$ | A1 | |
| | Obtain $\ldots + 2 + 5\frac{dy}{dx} = 0$ | B1 | B1 for $+2 + 5\frac{dy}{dx} = 0$, maybe implied by later work |
| | Substitute $x = 3$ and $y = 1$ to find value of their $\frac{dy}{dx}$ | *M1 | Dependent on at least one $\frac{dy}{dx}$ present |
| | Obtain $\frac{dy}{dx} = -\frac{2}{14}$ | A1 | |
| | Attempt equation of line through (3, 1) with gradient of normal | DM1 | Allow one sign error |
| | Obtain $y = 7x - 20$ or equivalent unsimplified | A1 | **FT** on their perpendicular gradient |
| | | 7 | |

109. 9709_s19_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use quotient rule to find first derivative or equivalent | *M1 | |
| | Obtain $\frac{dy}{dx}=\frac{3\ln x-3x\times\frac{1}{x}}{(\ln x)^2}$ or equivalent | A1 | Condone lack of brackets in denominator unless specifically simplified to $2\ln x$ |
| | Equate first derivative to zero and attempt value of $x$ from $\ln x=k$ oe | DM1 | Must get as far as $x=$ |
| | Obtain $x=e$ | A1 | Allow $e^1$ |
| | Obtain $y=3e$ | A1 | Allow $3e^1$<br>**SC1**: If $3\ln x-3x\times\frac{1}{x}=0$ seen with no reference to $\frac{dy}{dx}$, then allow M1 A1 then following marks<br>**SC2**: If denominator incorrect and numerator correct/reversed/added then max marks M0A0M1A1A1<br>**SC3**: If numerator reversed then max marks M1A0M1A1A1 |
| | | 5 | |

110. 9709_w19_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use quotient rule (or product rule) to find first derivative | *M1 | Must have correct $u$ and $v$ |
| | Obtain $-\frac{1}{x(1+\ln x)^2}$ or (unsimplified) equivalent | A1 | |
| | Use $y=4$ to obtain $\ln x=-\frac{1}{2}$ or exact equivalent for $x$ | B1 | |
| | Substitute for $x$ in their first derivative | DM1 | |
| | Obtain $-4e^{\frac{1}{2}}$ or exact equivalent | A1 | Must be simplified to contain a single exponential term |
| | | 5 | |

111. 9709_w19_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $-4y-4x\frac{dy}{dx}$ from use of the product rule | B1 | |
| | Differentiate $-2y^2$ to obtain $-4y\frac{dy}{dx}$ | B1 | |
| | Obtain $2x$, $=0$ with no extra terms | B1 | |
| | Rearrange to obtain expression for $\frac{dy}{dx}$ and substitute $x=-1$, $y=2$ | M1 | |
| | Obtain $\frac{dy}{dx}=\frac{2x-4y}{4x+4y}$ OE and hence $-\frac{5}{2}$ | A1 | |
| | | 5 | |
| (ii) | Equate numerator of derivative to zero to produce equation in $x$ and $y$ | M1 | |
| | Substitute into equation of curve to produce equation in $x$ or $y$ | M1 | |
| | Obtain $-6y^2=1$ or $-\frac{3}{2}x^2=1$ OE and conclude | A1 | |
| | | 3 | |
| (iii) | Use denominator of derivative equated to zero with equation of curve to produce equation in $x$ | M1 | |
| | Obtain $3x^2=1$ and hence $x=\pm\frac{1}{\sqrt{3}}$ | A1 | OE |
| | | 2 | |

112. 9709_w19_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Differentiate using the product rule | *M1 | Must have $u$ and $v$ correct in a correct formula with $\frac{du}{dx}=2$ and $\frac{dv}{dx}=me^{-\frac{1}{2}x}$ |
| | Obtain correct $2e^{-\frac{1}{2}x}-\frac{1}{2}e^{-\frac{1}{2}x}(2x+5)$ | A1 | OE |
| | Equate first derivative to zero and solve for $x$ | DM1 | Solution must come from linear terms |
| | Obtain $x=-\frac{1}{2}$ only | A1 | |
| | Obtain $4e^{\frac{1}{4}}$ or exact equivalent only | A1 | |
| | | 5 | |

113. 9709_w19_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $\frac{dx}{d\theta} = 6\cos 2\theta$ | B1 | |
| | Obtain $\frac{dy}{d\theta} = 4\sec^2 2\theta$ | B1 | |
| | Divide $\frac{dy}{d\theta}$ by $\frac{dx}{d\theta}$ with $\theta$ equated to $\frac{1}{6}\pi$ | M1 | |
| | Obtain $\frac{16}{3}$ or exact equivalent | A1 | Allow FT on A1 if $\frac{dx}{d\theta} = 3\cos 2\theta$ and $\frac{dy}{d\theta} = 2\sec^2 2\theta$ |
| | | 4 | |
| (ii) | Equate expression for $\frac{dy}{dx}$ to 2 with only one trigonometry ratio used | *M1 | Either $\cos 2\theta$ or $\sec 2\theta$ |
| | Obtain $\cos^3 2\theta = \frac{1}{3}$ or $\sec^3 = 3$ | A1 | |
| | Attempt correct steps to find a value of $\theta$ from $\cos^3 2\theta = m$, $0 < m < 1$ | DM1 | |
| | Obtain $\theta = 0.402$ and no others within the range | A1 | AWRT<br>**SC**: Allow FT if $\frac{dx}{d\theta} = 3\cos 2\theta$ and $\frac{dy}{d\theta} = 2\sec^2 2\theta$ |
| | | 4 | |

114. 9709_m18_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Differentiate using product rule | *M1 | Obtaining form $k_1 \sin\frac{1}{2}x + k_2 x\cos\frac{1}{2}x$ |
| | Obtain correct $4\sin\frac{1}{2}x + 2x\cos\frac{1}{2}x$ or unsimplified equivalent | A1 | |
| | Attempt equation of tangent with numerical value for gradient | DM1 | Dependent on first **M1** |
| | Obtain $y = 4x$ | A1 | |
| | | 4 | |

115. 9709_m18_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain expression for $\frac{dy}{dx}$ with numerator quadratic, denominator linear | M1 | Or equivalent where separate derivatives evaluated first when $t=3$ |
| | Obtain $\frac{3t^2-6t}{2t+4}$ | A1 | |
| | Identify $t=3$ at $P$ | B1 | |
| | Obtain $\frac{9}{10}$ or equivalent | A1 | |
| | | 4 | |
| (ii) | Equate first derivative to zero and obtain non-zero value of $t$ | M1 | |
| | Obtain $t=2$ | A1 | |
| | Substitute to obtain $(12,-4)$ | A1 | |
| | | 3 | |
| (iii) | Equate expression for gradient to $m$ and rearrange to confirm $3t^2-(2m+6)t-4m=0$ | B1 | AG; necessary detail needed |
| | Attempt solution of quadratic inequality or equation resulting from discriminant | M1 | |
| | Obtain critical values $-\sqrt{72}-9$ and $\sqrt{72}-9$ | A1 | Or exact equivalents |
| | Conclude $m\leqslant-\sqrt{72}-9$, $m\geqslant\sqrt{72}-9$ | A1 | Or exact equivalents |
| | | 4 | |

116. 9709_s18_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $\frac{dx}{d\theta}=-4\sin2\theta+3\cos\theta$ | B1 | B1 may be implied |
| | Use $\frac{dy}{dx}=\frac{dy}{d\theta}/\frac{dx}{d\theta}$ in terms of $\theta$ or with 1 already substituted | M1 | |
| | Obtain or imply $\frac{dy}{dx}=\frac{-3\sin\theta}{-4\sin2\theta+3\cos\theta}$ | A1 | |
| | Substitute 1 to obtain 1.25 | A1 | Or greater accuracy 1.252013… |
| | | 4 | |
| (ii) | Equate denominator of first derivative to zero | M1 | |
| | Use $\sin2\theta=2\sin\theta\cos\theta$ | A1 | |
| | Obtain $\sin\theta=\frac{3}{8}$ | A1 | |
| | | 3 | |

117. 9709_s18_MS_22 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Differentiate to obtain form $\frac{k_1}{2x+9}-\frac{k_2}{x}$ | M1 | |
| | Obtain correct $\frac{6}{2x+9}-\frac{2}{x}$ | A1 | |
| | Equate first derivative to zero and attempt solution to $x = ...$ | M1 | Dependent on previous M1 |
| | Obtain $x=9$ | A1 | |
| | | 4 | |
| (ii) | Use appropriate method for determining nature of stationary point | M1 | Second derivative or gradient or value of $y$ |
| | Conclude minimum with no errors seen | A1 | |
| | | 2 | |

118. 9709_s18_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use product rule to differentiate first term obtaining form $k_1 y^2 \frac{dy}{dx}\sin 2x + k_2 y^3 \cos 2x$ | M1 | |
| | Obtain correct $3y^2\frac{dy}{dx}\sin 2x + 2y^3\cos 2x$ | A1 | |
| | State $3y^2\frac{dy}{dx}\sin 2x + 2y^3\cos 2x + 4\frac{dy}{dx} = 0$ | A1 | |
| | Identify $x=0$, $y=2$ as relevant point | B1 | |
| | Find equation of tangent through $(0, 2)$ with numerical gradient | M1 | Dependent on previous M1 |
| | Obtain $y=-4x+2$ or equivalent | A1 | |
| | | 6 | |

119. 9709_w18_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use product rule to differentiate $y$ obtaining $k_1e^{2t}+k_2te^{2t}$ | M1 | |
| | Obtain correct $3e^{2t}+6te^{2t}$ | A1 | |
| | State derivative of $x$ is $1+\frac{1}{t+1}$ | B1 | |
| | Use $\frac{dy}{dx}=\frac{dy}{dt}/\frac{dx}{dt}$ with $t=0$ to find gradient | M1 | |
| | Obtain $y=\frac{3}{2}x$ or equivalent | A1 | |
| | | 5 | |
| (ii) | Equate $\frac{dy}{dx}$ or $\frac{dy}{dt}$ to zero and solve for $t$ | M1 | Allow full marks if correct solution is obtained but $\frac{dx}{dt}$ is incorrect |
| | Obtain $t=-\frac{1}{2}$ | A1 | |
| | Obtain $x=-1.19$ | A1 | |
| | Obtain $y=-0.55$ | A1 | |
| | | 4 | |

120. 9709_w18_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State expression of form $k_1\cos 2x+k_2\sin 2x$ | M1 | |
| | State correct $2\cos 2x-6\sin 2x$ | A1 | |
| | | 2 | |
| (ii) | State $R=\sqrt{40}$ or 6.324… | B1 FT | Following their derivative |
| | Use appropriate trigonometry to find $\alpha$ | M1 | |
| | Obtain 1.249… | A1 | Allow $\alpha$ in degrees at this point |
| | Equate their $R\cos(2x+\alpha)$ to 3 and find $\cos^{-1}(3\div R)$ | *M1 | |
| | Carry out correct process to find one value of $\alpha$ | M1 | Dependent on *M1, allow for -0.086…. |
| | Obtain 1.979 | A1 | |
| | Carry out correct process to find second value of $\alpha$ within the range | M1 | Dependent on *M1 |
| | Obtain 3.055 | A1 | Allow 3.056 |
| | | 8 | |

121. 9709_w18_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Differentiate to obtain $10\cos 2x$ | B1 | |
| | Differentiate to obtain $-6\sec^2 2x$ | B1 | |
| | Equate first derivative to zero and find value for $\cos^3 2x$ | M1 | |
| | Use correct process for finding $x$ from $\cos^3 2x = k$ | M1 | |
| | Obtain 0.284 nfww | A1 | Or greater accuracy |
| | | 5 | |

122. 9709_w18_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Obtain $6ye^{2x} + 3e^{2x}\frac{dy}{dx}$ as derivative of $3ye^{2x}$ | B1 | Allow unsimplified |
| | Obtain $2y\frac{dy}{dx}$ as derivative of $y^2$ | B1 | |
| | Obtain 4 as a derivative of $4x$ and zero as a derivative of 10 | B1 | Dependent B mark, must have at least one of the two previous B marks |
| | Substitute 0 and 2 to find gradient of curve | M1 | Dependent on at least one B1 |
| | Obtain $-\frac{16}{7}$ or $-2.29$ | A1 | Allow greater accuracy |
| | | 5 | |

123. 9709_m17_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use product rule for derivative of $x^2 \sin y$ | M1 | |
| | Obtain $2x\sin y + x^2 \cos y\frac{dy}{dx}$ | A1 | |
| | Obtain $-3\sin 3y\frac{dy}{dx}$ as derivative of $\cos 3y$ | B1 | |
| | Obtain $2x\sin y + x^2 \cos y\frac{dy}{dx} - 3\sin 3y\frac{dy}{dx} = 0$ | A1 | |
| | Substitute $x = 2, y = \frac{1}{2}\pi$ to find value of $\frac{dy}{dx}$ | M1 | dep $\frac{dy}{dx}$ occurring at least once |
| | Obtain $-\frac{4}{3}$ | A1 | from correct work only |
| | Total: | 6 | |

124. 9709_s17_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Differentiate $x$ and $y$ and form $\frac{dy}{dx}$ | M1 | |
| | Obtain $\dfrac{4t^3-6t^2+8t-12}{3t^2+6}$ | A1 | First 2 marks may be implied by an attempt at division |
| | Carry out division at least as far as $kt$ or equivalent | M1 | For **M1**, it must be division by a quadratic factor. Allow attempt at factorisation with same conditions as for division |
| | Obtain $\frac{4}{3}t$ | A1 | |
| | Obtain $\frac{4}{3}t-2$ with complete division shown and no errors seen | A1 | |
| | **Total:** | 5 | |
| (ii) | State or imply gradient of straight line is $\frac{1}{2}$ | B1 | Allow **B1** if $y=\dfrac{1}{2}x+\dfrac{9}{2}$ is seen |
| | Attempt value of $t$ from their $\frac{dy}{dx}$ = their negative reciprocal of gradient of line | M1 | |
| | Obtain $t=0$ and hence $(1,5)$ | A1 | |
| | **Total:** | 3 | |

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125. 9709_s17_MS_21 Q: 8

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Apply product rule to find first derivative | *M1 | |
| | Obtain $6x\ln\left(\frac{1}{6}x\right)+3x$ or equivalent | A1 | Allow unsimplified for **A1** |
| | Identify $x=6$ at $P$ | B1 | |
| | Substitute their value of $x$ at $P$ into attempt at first derivative | DM1 | dep *M |
| | Obtain 18 | A1 | |
| | **Total:** | 5 | |
| (ii) | Equate their first derivative to zero and attempt solution of equation of form $k\ln\left(\frac{1}{6}x\right)+m=0$ | *M1 | |
| | Obtain $x$–coordinate of form $a_1e^{a_2}$ | DM1 | dep *M |
| | Obtain $x=6e^{-\frac{1}{2}}$ or exact equivalent | A1 | |
| | Substitute exact $x$–value in the form $a_1e^{a_2}$ and attempt simplification to remove ln | M1 | |
| | Obtain $-54e^{-1}$ or exact equivalent | A1 | |
| | **Total:** | 5 | |

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126. 9709_s17_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Use quotient rule (or product rule) to find first derivative | M1 | |
| | Obtain $\frac{8xe^{4x}+10e^{4x}}{(2x+3)^2}$ or equivalent | A1 | |
| | Substitute $x=0$ to obtain gradient $\frac{10}{9}$ | A1 | |
| | Form equation of tangent through $\left(0,\frac{1}{3}\right)$ with numerical gradient | M1 | |
| | Obtain $10x-9y+3=0$ | A1 | |
| | **Total:** | **5** | |

127. 9709_s17_MS_22 Q: 8

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $\frac{dx}{dt}=2\sin 2t$ | B1 | |
| | Obtain $\frac{dy}{dt}=6\sin^2 t\cos t-9\cos^2 t\sin t$ | B1 | |
| | Use $\frac{dy}{dx}=\frac{dy}{dt}/\frac{dx}{dt}$ for their first derivatives | M1 | |
| | Use identity $\sin 2t=2\sin t\cos t$ | B1 | |
| | Simplify to obtain $\frac{3}{2}\sin t-\frac{9}{4}\cos t$ with necessary detail present | A1 | |
| | **Total:** | **5** | |
| (ii) | Equate $\frac{dy}{dx}$ to zero and obtain $\tan t=k$ | M1 | |
| | Obtain $\tan t=\frac{3}{2}$ or equivalent | A1 | |
| | Substitute value of $t$ to obtain coordinates $(2.38, 2.66)$ | A1 | |
| | **Total:** | **3** | |
| (iii) | Identify $t=\frac{1}{4}\pi$ | B1 | |
| | Substitute to obtain exact value for gradient of the normal | M1 | |
| | Obtain gradient $\frac{4}{3}\sqrt{2}$ , $\frac{8}{3\sqrt{2}}$ or similarly simplified exact equivalent | A1 | |
| | **Total:** | **3** | |

128. 9709_w17_MS_21 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $\frac{dx}{dt} = 4e^{2t} + 4e^t$ | **B1** | |
| | Use product rule to find $\frac{dy}{dt}$ | **M1** | |
| | Obtain $\frac{dy}{dx} = \frac{5e^{2t} + 10te^{2t}}{4e^{2t} + 4e^t}$ or equivalent | **A1** | |
| | Equate first derivative of the form $\frac{ae^{2t} + bte^{2t}}{ce^{2t} + de^t}$ to zero and solve to find $t$ | **M1** | |
| | Obtain $t = -\frac{1}{2}$ from completely correct work | **A1** | |
| | Obtain $(3.16, -0.92)$ | **A1** | |
| | | **6** | |
| (ii) | Identify $t = 0$ | **B1** | |
| | Substitute $t = 0$ in expression for first derivative and find negative reciprocal | **M1** | |
| | Obtain $-\frac{8}{5}$ or equivalent | **A1** | |
| | | **3** | |

129. 9709_w17_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Differentiate to obtain form $k_1 \sec^2 \frac{1}{2}x + k_2 \cos \frac{1}{2}x$ | **M1** | If a factor of 0.5 is missed, can still get 5/6, penalise at first **A1** |
| | Obtain $\frac{1}{2}\sec^2 \frac{1}{2}x + \frac{3}{2}\cos \frac{1}{2}x$ | **A1** | |
| | Equate first derivative to zero and produce $\cos^3 \frac{1}{2}x = k_3$ | ***M1** | |
| | Use correct process to find one value of $x$ | **DM1** | Dep on *M, allow for obtaining 1.609….. , 92.2° or 268° |
| | Obtain $x = 4.67$ | **A1** | Allow $x = 4.67$ or better for **A1** |
| | Obtain $y = 1.12$ | **A1** | Allow $y = 1.12$ from $x = 4.66$ but nothing else |
| | | **6** | |

130. 9709_w17_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain $4y+4x\frac{dy}{dx}$ as derivative of $4xy$ | **B1** | |
| | Obtain $4y\frac{dy}{dx}$ as derivative of $2y^2$ | **B1** | |
| | State $2x+4y+4x\frac{dy}{dx}+4y\frac{dy}{dx}=0$ | **B1** | 3rd **B1** may be implied by later work |
| | Substitute $x=-1,\ y=3$ to find gradient of line | ***M1** | dep at least one **B1** |
| | Form equation of tangent through $(-1, 3)$ with numerical gradient | **DM1** | dep *M |
| | Obtain $5x+4y-7=0$ or equivalent of required form | **A1** | Allow any 3 term integer form for **A1** |
| | | **6** | |
| (ii) | Substitute $\frac{dy}{dx}=\frac{1}{2}$ to find relation between $x$ and $y$ | ***M1** | dep at least one **B1** in part **(i)**, must be linear |
| | Obtain $4x+6y=0$ or equivalent | **A1** | |
| | Substitute for $x$ or $y$ in equation of curve | **DM1** | dep on *M |
| | Obtain $-\frac{7}{4}y^2=7$ or $-\frac{7}{9}x^2=7$ or equivalent and conclude appropriately | **A1** | |
| | | **4** | |

131. 9709_m16_MS_22 Q: 6

| | | | |
|---|---|---|---|
| **(i)** | Use product rule to obtain expression of form $k_1e^{-x}\sin 2x+k_2e^{-x}\cos 2x$ | **M1** | |
| | Obtain correct $-3e^{-x}\sin 2x+6e^{-x}\cos 2x$ | **A1** | |
| | Substitute $x=0$ in first derivative to obtain equation of form $y=mx$ | **M1** | |
| | Obtain $y=6x$ or equivalent with no errors in solution | **A1** | [4] |
| **(ii)** | Equate first derivative to zero and obtain $\tan 2x=k$ | **M1*** | |
| | Carry out correct process to find value of $x$ | **dep M1*** | |
| | Obtain $x=0.554$ | **A1** | |
| | Obtain $y=1.543$ | **A1** | [4] |

132. 9709_m16_MS_22 Q: 7

| | | | |
|---|---|---|---|
| **(i)** | State $3y^2\frac{dy}{dx}$ as derivative of $y^3$ | **B1** | |
| | Equate derivative of left-hand side to zero and solve for $\frac{dy}{dx}$ | **M1** | |
| | Obtain $\frac{dy}{dx} = -\frac{6x^2}{3y^2}$ or equivalent | **A1** | |
| | Observe $x^2$ and $y^2$ never negative and conclude appropriately | **A1** | [4] |
| **(ii)** | Equate first derivative to $-2$ and rearrange to $y^2 = x^2$ or equivalent | **B1** | |
| | Substitute in original equation to obtain at least one equation in $x^3$ or $y^3$ | **M1** | |
| | Obtain $3x^3 = 24$ or $x^3 = 24$ or $3y^3 = 24$ or $-y^3 = 24$ | **A1** | |
| | Obtain $(2, 2)$ | **A1** | |
| | Obtain $(\sqrt[3]{24}, -\sqrt[3]{24})$ or $(2.88, -2.88)$ and no others | **A1** | [5] |

133. 9709_s16_MS_21 Q: 1

| | | |
|---|---|---|
| Obtain first derivative of form $k_1e^{4x} + \frac{k_2}{2x+3}$ | **M1** | |
| Obtain correct $12e^{4x} - \frac{12}{2x+3}$ | **A1** | |
| Obtain 8 | **A1** | [3] |

134. 9709_s16_MS_21 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | Obtain $\frac{dx}{d\theta} = 2\sec^2\theta$ and $\frac{dy}{d\theta} = 6\cos 2\theta$ | **B1** | |
| | Use $\cos 2\theta = 2\cos^2\theta - 1$ or equivalent | **B1** | |
| | Form expression for $\frac{dy}{dx}$ in terms of $\cos\theta$ | **M1** | |
| | Confirm $6\cos^4\theta - 3\cos^2\theta$ with no errors seen | **A1** | [4] |
| **(ii)** | Equate first derivative to zero and obtain at least $\cos\theta = \pm\frac{1}{\sqrt{2}}$ | **B1** | |
| | Obtain $\theta = \frac{1}{4}\pi$ or equivalent | **B1** | |
| | Obtain $(2, 3)$ | **B1** | [3] |
| **(iii)** | State or imply $\theta = \frac{1}{3}\pi$ or equivalent | **B1** | |
| | Obtain $-\frac{3}{8}$ or equivalent only | **B1** | [2] |

135. 9709_s16_MS_22 Q: 7

| | | | |
|---|---|---|---|
| **(i)** | State $\frac{dx}{dt} = \sin t$ and $\frac{dy}{dt} = -6\sin 2t$ | **B1** | |
| | Use $\sin 2t = 2\sin t \cos t$ | **B1** | |
| | Form expression for $\frac{dy}{dx}$ in terms of $t$ | **M1** | |
| | Confirm $-12\cos t$ | **A1** | [4] |
| **(ii)** | Identify $\frac{1}{2}\pi$ as value of $t$ | **B1** | |
| | Obtain $(2, -2)$ | **B1** | [2] |
| **(iii)** | Identify $\cos 2t = -\frac{1}{3}$ | **B1** | |
| | Attempt to find value of $t$ (or of $\cos t$) for at least one of the two points | **M1** | |
| | Obtain 0.955 (or $\frac{1}{\sqrt{3}}$) or 2.186 (or $-\frac{1}{\sqrt{3}}$) | **A1** | |
| | Obtain $-\frac{12}{\sqrt{3}}$ or $-4\sqrt{3}$ or $-6.93$ and $\frac{12}{\sqrt{3}}$ or $4\sqrt{3}$ or 6.93 | **A1** | [4] |

136. 9709_w16_MS_21 Q: 3

| | | | |
|---|---|---|---|
| | Differentiate to obtain $4\cos 2x + 10\sin 2x$ | **B1** | |
| | Equate first derivative to zero and arrange to $\tan 2x = \dots$ | ***M1** | |
| | Obtain $\tan 2x = -0.4$ | **A1** | |
| | Carry out correct method for finding at least one value of $x$, dependent *M | **DM1** | |
| | Obtain $x = 1.38$ | **A1** | |
| | Obtain $x = 2.95$ and no others between 0 and $\pi$ | **A1** | [6] |

137. 9709_w16_MS_21 Q: 6

| | | | |
|---|---|---|---|
| | Differentiate $4xy$ to obtain $4y + 4x\frac{dy}{dx}$ | **B1** | |
| | Differentiate $y^2$ to obtain $2y\frac{dy}{dx}$ | **B1** | |
| | Equate attempt of derivative of left-hand side to zero | **M1** | |
| | Substitute $(1, 3)$ to find numerical value of derivative | **M1** | |
| | Obtain $-\frac{18}{10}$ or $-\frac{9}{5}$ | **A1** | |
| | Obtain $\frac{10}{18}$ or $\frac{5}{9}$ as gradient of normal, following their numerical value of derivative | **A1**√ | |
| | Form equation of normal at $(1, 3)$ | **M1** | |
| | Obtain $5x - 9y + 22 = 0$ or equivalent of requested form | **A1** | [8] |

138. 9709_s15_MS_21 Q: 3

| | | |
|---|---|---|
| Differentiate to obtain form $p\cos x + q\sin 2x$ or equivalent | M1 | |
| Obtain correct $6\cos x + 4\sin 2x$ or equivalent | A1 | |
| Substitute $\frac{1}{6}\pi$ to obtain derivative equal to $5\sqrt{3}$ or 8.66 | A1 | |
| Form equation of tangent (not normal) using numerical value of gradient obtained by differentiation | M1 | |
| Obtain $y = 8.66x - 2.53$ cao | A1 | [5] |

139. 9709_s15_MS_21 Q: 7

| | | | |
|---|---|---|---|
| **(i)** | Obtain $3y^2\frac{dy}{dx}$ as derivative of $y^3$ | B1 | |
| | Obtain $4y + 4x\frac{dy}{dx}$ as derivative of $4xy$ | B1 | |
| | Equate derivative of left-hand side to zero and solve for $\frac{dy}{dx}$, must be from implicit differentiation | M1 | |
| | Confirm given answer $\frac{dy}{dx} = -\frac{4y}{3y^2 + 4x}$ correctly | A1 | [4] |
| **(ii)** | State or imply $y = 0$ | B1 | |
| | Substitute in equation of curve and show contradiction | B1 | [2] |
| **(iii)** | State or imply $3y^2 + 4x = 0$ | B1 | |
| | Eliminate one variable from equation of curve using $3y^2 + 4x = 0$ | M1 | |
| | Obtain $y = -2$ | A1 | |
| | Obtain $x = -3$ | A1 | [4] |

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140. 9709_w15_MS_21 Q: 2

| | | |
|---|---|---|
| Use quotient rule or, after adjustment, product rule | **M1*** | |
| Obtain $\frac{3x - 15 - 3x - 1}{(x-5)^2}$ or equivalent | **A1** | |
| Equate first derivative to –4 and solve for $x$ | **M1 dep** | |
| Obtain $x$-coordinates 3 and 7 or one correct pair of coordinates | **A1** | |
| Obtain $y$-coordinates –5 and 11 respectively or other correct pair of coordinates | **A1** | [5] |

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141. 9709_w15_MS_21 Q: 7

| | | | |
|---|---|---|---|
| **(i)** | Obtain $12\sin t\cos t$ or equivalent for $\frac{dx}{dt}$ | **B1** | |
| | Obtain $4\cos 2t - 6\sin 2t$ or equivalent for $\frac{dy}{dt}$ | **B1** | |
| | Obtain expression for $\frac{dy}{dx}$ in terms of $t$ | **M1** | |
| | Use $2\sin t\cos t = \sin 2t$ | **A1** | |
| | Confirm given answer $\frac{dy}{dx} = \frac{2}{3}\cot 2t - 1$ with no errors seen | **A1** | [5] |
| **(ii)** | State or imply $\tan 2t = \frac{2}{3}$ | **B1** | |
| | Obtain $t = 0.294$ | **B1** | |
| | Obtain $t = 1.865$ | **B1** | [3] |
| **(iii)** | Attempt solution of $2\sin 2t + 3\cos 2t = 0$ at least as far as $\tan 2t = \ldots$ | **M1** | |
| | Obtain $\tan 2t = -\frac{3}{2}$ or equivalent | **A1** | |
| | Substitute to obtain $-\frac{13}{9}$ | **A1** | [3] |

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142. 9709_w15_MS_22 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | Use product rule to obtain form $k_1 e^{-3x} + k_2 x e^{-3x}$ | **M1** | |
| | Obtain correct $4e^{-3x} - 12xe^{-3x}$ | **A1** | |
| | Obtain $x = \frac{1}{3}$ or 0.333 or better and no other | **A1** | [3] |
| **(ii)** | Use quotient rule or equivalent | **M1*** | |
| | Obtain correct numerator $8x(x+1) - 4x^2$ or equivalent | **A1** | |
| | Equate numerator to zero and solve to find at least one value | **M1 dep** | |
| | Obtain $x = -2$ | **A1** | |
| | Obtain $x = 0$ | **A1** | [5] |

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143. 9709_w15_MS_22 Q: 6

| | | | | |
|---|---|---|---|---|
| **(i)** | Either | Obtain $\frac{dx}{dt} = -3\sin t$ | **B1** | |
| | | Obtain $\frac{dy}{dt} = -2\sin(t - \frac{1}{6}\pi)$ | **B1** | |
| | | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** | |
| | | Expand $-2\sin(t - \frac{1}{6}\pi)$ to obtain $k_1 \sin t + k_2 \cos t$ | **M1** | |
| | | Confirm given result $\frac{1}{3}(\sqrt{3} - \cot t)$ correctly | **A1** | |
| | Or | Obtain $\frac{dx}{dt} = -3\sin t$ | **B1** | |
| | | Expand $y$ to obtain $k_3 \cos t + k_4 \sin t$ | **M1** | |
| | | Obtain $\frac{dy}{dt} = -\sqrt{3}\sin t + \cos t$ or equivalent | **A1** | |
| | | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** | |
| | | Confirm given result $\frac{1}{3}(\sqrt{3} - \cot t)$ correctly | **A1** | [5] |
| **(ii)** | | Identify value of $t$ as $\frac{1}{2}\pi$ only | **B1** | |
| | | Obtain gradient at relevant point as $\frac{1}{3}\sqrt{3}$ or 0.577 or better | **B1** | |
| | | Form equation of tangent through (0, 1), using their gradient | **M1** | |
| | | Obtain $y = \frac{1}{3}\sqrt{3}x + 1$ or equivalent | **A1** | [4] |

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144. 9709_w15_MS_23 Q: 3

| | | |
|---|---|---|
| Obtain $\frac{dx}{dt} = e^t + (t+1)e^t$ or equivalent | **B1** | |
| Obtain $\frac{dy}{dt} = t(t+4)^{-\frac{1}{2}}$ | **B1** | |
| Substitute $t = 0$ and divide to obtain gradient of tangent | **M1** | |
| Obtain $\frac{3}{4}$ following their first derivatives | **A1** | |
| Form equation of tangent through $(1, 12)$ | **M1** | |
| Obtain $3x - 4y + 45 = 0$ or equivalent of required form | **A1** | [6] |

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145. 9709_w15_MS_23 Q: 7

**(i)** Use quotient rule or equivalent to find first derivative **M1**

Obtain $\dfrac{2\cos 2x(\cos x+1)+\sin 2x\sin x}{(\cos x+1)^2}$ or equivalent **A1**

Use at least one of $\cos 2x = 2\cos^2 x - 1$ and $2x = 2\sin x\cos x$ **B1**

Express first derivative in terms of $\cos x$ only **M1**

Obtain $\dfrac{2\cos^3 x+4\cos^2 x-2}{(\cos x+1)^2}$ or equivalent **A1**

Factorise numerator or divide numerator by $(\cos x+1)$ or equivalent **M1**

Confirm given answer $\dfrac{2(\cos^2 x+\cos x-1)}{\cos x+1}$ correctly **A1** [7]

**(ii)** Use quadratic formula or equivalent to find value of $\cos x$ **M1**

Obtain $x$-coordinate 0.905 **A1**

Obtain $x$-coordinate –0.905 and no others in range **A1** [3]

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146. 9709_s20_MS_21 Q: 6

| | | |
|---|---|---|
| (a) | Express left-hand side in terms of $\sin\theta$ and $\cos\theta$ | M1 |
| | Obtain $2\cos\theta - 2\sin\theta$ | A1 |
| | Attempt to express $a\cos\theta + b\sin\theta$ in $R\cos(\theta+\beta)$ form | M1 |
| | Confirm $R=\sqrt{8}$ AG | A1 |
| | Carry out necessary trigonometry and confirm $\frac{1}{4}\pi$ AG | A1 |
| | | 5 |
| (b) | Carry out correct process to find $\theta$ from $\cos\left(\theta+\frac{1}{4}\pi\right)=\frac{1}{\sqrt{8}}$ | M1 |
| | Obtain 0.424 | A1 |
| | | 2 |
| (c) | Express integrand as $\sqrt{8}\cos\left(\frac{1}{2}x+\frac{1}{4}\pi\right)$ or as $2\cos\frac{1}{2}x - 2\sin\frac{1}{2}x$ | B1 |
| | Integrate to obtain $k\sin\left(\frac{1}{2}x+\frac{1}{4}\pi\right)$ or $k_1\sin\frac{1}{2}x + k_2\cos\frac{1}{2}x$ | M1 |
| | Obtain correct $2\sqrt{8}\sin\left(\frac{1}{2}x+\frac{1}{4}\pi\right)$ or $4\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$ | A1 |
| | | 3 |

147. 9709_s20_MS_21 Q: 7

| | | |
|---|---|---|
| (a) | Carry out division at least as far as $3x^2 + kx$ | M1 |
| | Obtain quotient $3x^2 - 4x - 4$ | A1 |
| | Confirm remainder is 9 AG | A1 |
| | | 3 |
| (b) | Integrate to obtain at least $k_1 x^3$ and $k_2 \ln(3x+2)$ terms | *M1 |
| | Obtain $x^3 - 2x^2 - 4x + 3\ln(3x+2)$<br>(**FT** from quotient in part **(a)**) | A1FT |
| | Apply limits correctly | DM1 |
| | Apply appropriate logarithm properties correctly | M1 |
| | Obtain $125 + \ln 64$ | A1 |
| | | 5 |
| (c) | State or imply $9x^3 - 6x^2 - 20x - 8 = (3x+2)(3x^2 - 4x - 4)$<br>(**FT** from quotient in part **(a)**) | B1FT |
| | Attempt to solve cubic eqn to find positive value of $x$ (or of $e^{3y}$ ) | M1 |
| | Use logarithms to solve equation of form $e^{3y} = k$ where $k > 0$ | M1 |
| | Obtain $\frac{1}{3}\ln 2$ or exact equivalent | A1 |
| | | 4 |

148. 9709_s20_MS_22 Q: 7

| | | |
|---|---|---|
| (a) | Integrate to obtain the form $k_1 \ln(2x+1) + k_2 x^2$ | *M1 |
| | Obtain correct $2\ln(2x+1) + 4x^2$ | A1 |
| | Use limits correctly and attempt rearrangement | DM1 |
| | Confirm $a = \sqrt{2.5 - 0.5\ln(2a+1)}$ AG | A1 |
| | | 4 |
| (b) | Consider sign of $a - \sqrt{2.5 - 0.5\ln(2a+1)}$ or equivalent for 1 and 2 | M1 |
| | Obtain $-0.3...$ and $0.6...$ or equivalents and justify conclusion | A1 |
| | | 2 |
| (c) | Use iteration process correctly at least once | M1 |
| | Obtain final answer 1.358 | A1 |
| | Show sufficient iterations to 6 sf to justify answer or show a sign change in the interval [1.3575, 1.3585] | A1 |
| | | 3 |

149. 9709_s20_MS_22 Q: 8

| | | |
|---|---|---|
| (a) | Use at least one of $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ | **B1** |
| | Use both and conclude $6\cos^2\theta$ AG | **B1** |
| | | **2** |
| (b) | Attempt solution of $\cos^2\theta = \dfrac{5}{6}$ to find at least one value | **M1** |
| | Obtain 0.421 | **A1** |
| | Obtain 2.72 | **A1** |
| | | **3** |
| (c) | Express integrand in form $a + b\cos x$ | **M1** |
| | Obtain correct integrand $3 + 3\cos x$ | **A1** |
| | Integrate to obtain $px + q\sin x$ | ***M1** |
| | Apply limits correctly | **DM1** |
| | Obtain $\dfrac{3}{4}\pi + 3 - \dfrac{3}{\sqrt{2}}$ or exact equivalent | **A1** |
| | | **5** |

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150. 9709_w20_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Integrate to obtain form $ax + be^{-2x}$ | **M1** | |
| | Obtain correct $2x - \frac{1}{2}e^{-2x}$ | **A1** | |
| | Apply limits to obtain $\frac{5}{2} - \frac{1}{2}e^{-2}$ | **A1** | |
| | Attempt to find area of relevant trapezium | **M1** | |
| | Obtain $\frac{5}{2} + \frac{1}{2}e^{-2}$ and subtract to obtain $e^{-2}$ or exact equivalent | **A1** | |
| | | **5** | |

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151. 9709_w20_MS_21 Q: 8

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Differentiate using the quotient rule (or product rule) | *M1 | |
| | Obtain $\frac{(2x-1)(12x^2+8)-2(4x^3+8x-4)}{(2x-1)^2}$ | A1 | OE |
| | Equate first derivative to zero and attempt solution | DM1 | |
| | Obtain $(0, 4)$ | A1 | Allow if given separately<br>Allow A1 if both $x$-coordinates are given, but $y$ coordinates are omitted. |
| | Obtain $\left(\frac{3}{4}, \frac{59}{8}\right)$ | A1 | |
| | | 5 | |
| (b) | Carry out division to obtain quotient of form $2x^2+kx+m$ | M1 | For non-zero constants $k$, $m$ |
| | Obtain correct quotient $2x^2+x+\frac{9}{2}$ | A1 | |
| | Obtain remainder $\frac{1}{2}$ | A1 | |
| | Integrate to obtain at least $k_1x^3$ and $k_2\ln(2x-1)$ terms | M1 | For non-zero constants $k_1$, $k_2$ |
| | Obtain $\frac{2}{3}x^3+\frac{1}{2}x^2+\frac{9}{2}x+\frac{1}{4}\ln(2x-1)$ as final answer | A1 | Condone absence of ...$+c$ and modulus signs |
| | | 5 | |

152. 9709_w20_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Differentiate using quotient rule (or product rule) | *M1 | |
| | Obtain $\frac{(x^2+8)-2x(x-2)}{(x^2+8)^2}$ | A1 | OE |
| | Equate first derivative to zero and attempt solution to get $x = \ldots$ | DM1 | |
| | Obtain $2\pm\sqrt{12}$ or exact equivalents | A1 | |
| | | 4 | |
| (b) | Use $y$ values $(0), \frac{4}{44}, \frac{8}{108}, \frac{12}{204}$ or decimal equivalents | B1 | Decimal equivalents need to be to at least 2 decimal places |
| | Use correct formula, or equivalent, with $h=4$ | M1 | |
| | Obtain $2\left(0+2\times\frac{4}{44}+2\times\frac{8}{108}+\frac{12}{204}\right)$ or equivalent and hence 0.78 | A1 | |
| | | 3 | |

153. 9709_w20_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Express $\frac{8}{\cos^2(4x+1)}$ as $8\sec^2(4x+1)$ | B1 | SOI |
| | Integrate to obtain the form $a\ln(4x+1)$ | M1 | |
| | Integrate to obtain $b\tan(4x+1)$ | M1 | |
| | Obtain $2\ln(4x+1)+2\tan(4x+1)+c$ | A1 | Condone use of brackets rather than modulus signs |
| | | 4 | |
| (b) | Express $4\cos^2\frac{1}{2}x$ in the form $p+q\cos x$ | M1 | For constants where $pq \neq 0$ |
| | Obtain correct $2+2\cos x$ | A1 | |
| | Integrate to obtain form $px+q\sin x+r\cos 2x$ | *M1 | For constants where $pqr \neq 0$ |
| | Obtain correct $5x+2\sin x-\frac{1}{2}k\cos 2x$ | A1 | Allow $3x + 2x$ in place of $5x$ |
| | Apply limits correctly, equate to 10 and solve for $k$ | DM1 | |
| | Obtain $k=8-\frac{5}{2}\pi$ | A1 | CWO |
| | | 6 | |

154. 9709_m19_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Integrate to obtain form $k_1\ln x+k_2\ln(2x+1)$ | M1 | |
| | Obtain correct $2\ln x+\ln(2x+1)$ | A1 | |
| | Use logarithm addition/subtraction property correctly | M1 | |
| | Use logarithm power property correctly | M1 | |
| | Confirm $\ln 48$ with no errors seen | A1 | Answer given; necessary detail needed |
| | | 5 | |
| (b) | Use identity $\sin 2x = 2\sin x\cos x$ | B1 | |
| | State or imply $\cot x+2\operatorname{cosec} x=\frac{\cos x}{\sin x}+\frac{2}{\sin x}$ | B1 | |
| | Attempt to express integrand in terms of $\cos 2x$ and $\cos x$ | M1 | |
| | Obtain correct integrand $1+\cos 2x+4\cos x$ | A1 | |
| | Integrate to obtain at least terms $k_3\sin 2x$ and $k_4\sin x$ | M1 | |
| | Obtain correct $x+\frac{1}{2}\sin 2x+4\sin x+c$ | A1 | |
| | | 6 | |

155. 9709_s19_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Use identity $\tan^2 3x = \sec^2 3x - 1$ | B1 | |
| | Integrate to obtain form $k_1 \tan 3x + k_2 x$ | M1 | |
| | Obtain correct $\frac{1}{3}\tan 3x - x + c$ | A1 | |
| | | 3 | |
| (b) | Express integrand as $e^{2x} + 4e^{-x}$ | B1 | |
| | Integrate to obtain form $k_3 e^{2x} + k_4 e^{-x}$ | M1 | |
| | Obtain correct $\frac{1}{2}e^{2x} - 4e^{-x}$ | A1 | |
| | Use limits to obtain $\frac{1}{2}e^2 - 4e^{-1} + \frac{7}{2}$ or similarly simplified equivalent | A1 | |
| | | 4 | |

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156. 9709_s19_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Use identity $2\cos^2 x = 1 + \cos 2x$ | B1 | |
| | Integrate to obtain form $x + \frac{1}{2}\sin 2x$ | B1 | |
| | Integrate to obtain $-2\cos 2x$ | B1 | |
| | Apply limits correctly, retaining exactness | M1 | Dependent on at least one B mark |
| | Obtain $4 + \frac{1}{2}\pi$ or similarly simplified equivalent | A1 | |
| | | 5 | |
| (b) | Use $y$ values $\sqrt{\ln 3}$, $\sqrt{\ln 6}$, $\sqrt{\ln 9}$ or decimal equivalents | B1 | Allow awrt 1.05, 1.34, 1.48, the correct level of accuracy may be implied by a correct answer |
| | Use correct formula, or equivalent, with $h = 3$, and three $y$ values | M1 | |
| | Obtain $\frac{1}{2} \times 3\left(\sqrt{\ln 3} + 2\sqrt{\ln 6} + \sqrt{\ln 9}\right)$ and hence 7.81 | A1 | Allow greater accuracy |
| | | 3 | |

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157. 9709_s19_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Carry out division to obtain quotient of form $x^2+k$ | M1 | |
| | Obtain quotient $x^2-4$ | A1 | Allow use of an identity |
| | Obtain remainder 4 | A1 | |
| | | 3 | SC: If only the remainder theorem is used to obtain 4 then B1 |
| (ii) | Integrate to obtain at least $k_1x^3$ and $k_2\ln(2x+1)$ terms using the result from **(i)** | *M1 | |
| | Obtain correct $\frac{1}{3}x^3-4x+2\ln(2x+1)$ | A1 | |
| | Apply limits and note or imply that constant $k_3$ can be written $\ln e^{k_3}$ | DM1 | |
| | Apply appropriate logarithm properties correctly | M1 | |
| | Obtain $\ln(49e^{-3})$ | A1 | |
| | | 5 | |

158. 9709_w19_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Expand integrand to obtain $4e^{4x}-4e^{2x}+1$ | B1 | |
| | Integrate to obtain at least two terms of form $k_1e^{4x}+k_2e^{2x}+k_3x$ | *M1 | |
| | Obtain correct $e^{4x}-2e^{2x}+x$ | A1 | |
| | Apply both limits correctly to their integral | DM1 | |
| | Obtain $e^8-3e^4+2e^2+1$ | A1 | |
| | | 5 | |

159. 9709_w19_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Obtain $\frac{3}{2}\ln x$ or $\frac{3}{2}\ln(2x)$ or $\frac{3}{2}\ln(kx)$ | B1 | |
| | Use subtraction law of logarithms correctly, showing sufficient detail | M1 | $\ln 216 - \ln 8 = \ln\left(\frac{216}{8}\right)$ |
| | Use power law of logarithms correctly | M1 | $n\ln(kx) = \ln(kx)^n$ |
| | Confirm $\ln 27$ with sufficient working and no incorrect working | A1 | AG |
| | | 4 | |
| (b) | Use appropriate identity to express integrand in form $k_1 + k_2\cos 3x$ | *M1 | $k_1 \neq 0$. Allow $2\times\frac{3}{2}x$ for $3x$ |
| | Obtain correct $2 - 2\cos 3x$ | A1 | |
| | Integrate to obtain form $k_3x + k_4\sin 3x$ | DM1 | |
| | Obtain correct $2x - \frac{2}{3}\sin 3x$ | A1 | |
| | Use limits to obtain $\frac{1}{3}\pi - \frac{2}{3}$ or exact equivalent | A1 | |
| | | 5 | |

160. 9709_m18_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use $y$-values $\ln 2$, $\ln 4$, $\ln 6$, $\ln 8$, $\ln 10$ | B1 | Or decimal equivalents |
| | Use correct formula, or equivalent, with $h = 2$ and five $y$-values | M1 | |
| | Obtain 13.5 | A1 | |
| | | 3 | |
| (ii) | Recognise integrand as $6\ln(x+2)$ | B1 | |
| | Obtain 81 or 81.0 or 81.1 | B1 | |
| | | 2 | |

161. 9709_m18_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Express LHS in terms of $\sin 2x$ and $\cos 2x$ and attempt to express in terms of $\sin x$ and $\cos x$ | *M1 | |
| | Obtain correct $\frac{1}{2\sin x\cos x}+\frac{\cos^2 x-\sin^2 x}{2\sin x\cos x}$ or equivalent | A1 | Perhaps using $\cos 2x = 2\cos^2 x - 1$ immediately |
| | Simplify as far as single terms involving $x$ in numerator and denominator | DM1 | Dependent on first M mark |
| | Confirm $\cot x$ | A1 | AG; necessary detail needed |
| | | 4 | |
| (ii) | Express in terms of $\sin\frac{1}{6}\pi$ and $\cos\frac{1}{6}\pi$ <u>or</u> $\sin\frac{1}{6}\pi$ and $\tan\frac{1}{6}\pi$ | M1 | |
| | Obtain $2+\sqrt{3}$ | A1 | |
| | | 2 | |
| (iii) | State $\int \sin 2x \cot 2x \, dx$ | B1 | Condoning absence of d*x* |
| | State $\int \cos 2x \, dx$ | B1 | Condoning absence of d*x* |
| | Obtain $\frac{1}{2}\sin 2x + c$ | B1 | |
| | | 3 | |

162. 9709_s18_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Rewrite integrand as $4e^{2x}+4e^{-x}$ | B1 | |
| | Integrate to obtain form $k_1e^{2x}+k_2e^{-x}$ where $k_1 \neq 4, k_2 \neq 4$ | M1 | |
| | Obtain correct $2e^{2x}-4e^{-x}$ | A1 | |
| | Apply limits correctly, retaining exactness | M1 | Dependent on previous M1 |
| | Obtain $2e^4-4e^{-2}+2$ or exact similarly simplified equivalent | A1 | |
| | | 5 | |

163. 9709_s18_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State $R = \sqrt{29}$ or 5.385… | B1 | |
| | Use appropriate trigonometry to find $\alpha$ | M1 | Allow M1 for $\tan\alpha = \pm\frac{2}{5}$ or $\pm\frac{5}{2}$ oe |
| | Obtain 0.3805 with no errors seen | A1 | Or greater accuracy 0.3805063… |
| | | 3 | |
| (ii) | State that equation is $5\cos\theta - 2\sin\theta = 4$ | B1 | |
| | Evaluate $\cos^{-1}(k/R) - \alpha$ to find one value of $\theta$ | M1 | Allow M1 from their $\sqrt{29}\cos(\theta \pm \alpha)$ |
| | Obtain 0.353 | A1 | Or greater accuracy 0.35307… |
| | Carry out correct method to find second value | M1 | |
| | Obtain 5.17 and no extra solutions in the range | A1 | Or greater accuracy 5.16909…<br>If working consistently in degrees, then no A marks are available, B1, M1, M1 max |
| | | 5 | |
| (iii) | State integrand as $\frac{1}{29}\sec^2(\frac{1}{2}x + 0.3805)$ | B1 FT | Following their answer from part (i), must be in the form $R\cos(\theta \pm \alpha)$ |
| | Integrate to obtain form $k\tan(\frac{1}{2}x + \text{their } \alpha)$ | M1 | |
| | Obtain $\frac{2}{29}\tan(\frac{1}{2}x + 0.3805) + c$ | A1 | |
| | | 3 | |

164. 9709_s18_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Express $\operatorname{cosec}^2 2x$ as $\dfrac{1}{4\sin^2 x\cos^2 x}$ | B1 | |
| | Attempt to express LHS in terms of $\sin x$ and $\cos x$ only | M1 | Must be using correct working for M1 |
| | Obtain $\dfrac{2\times 2\sin^2 x}{4\sin^2 x\cos^2 x}$ or equivalent and hence $\sec^2 x$ | A1 | AG; necessary detail needed |
| | | 3 | |
| (ii) | Express equation as $1 + \tan^2 x = \tan x + 21$ | B1 | |
| | Solve 3-term quadratic equation for $\tan x$ | M1 | |
| | Obtain $\tan x = 5$ and hence $x = 1.37$ | A1 | Or greater accuracy 1.3734… |
| | Obtain $\tan x = -4$ and hence $x = 1.82$ | A1 | Or greater accuracy 1.8157… |
| | | 4 | |
| (iii) | Use $x = 2y + 1$ | B1 | |
| | Identify integral as of form $\int \sec^2(ay + b)\,dy$ | M1 | Condone absence of or error with $dy$ |
| | Obtain $\frac{1}{2}\tan(2y + 1) + c$ | A1 | |
| | | 3 | |

165. 9709_w18_MS_21 Q: 2

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Integrate to obtain form $k\ln(2x+1)$ | M1 | |
| | Obtain correct $3\ln(2x+1)$ | A1 | |
| | Use subtraction law of logarithms correctly | M1 | Dependent on first M1 |
| | Use power law of logarithms correctly | M1 | Dependent on first M1 |
| | Confirm $\ln 125$ | A1 | |
| | | 5 | |

166. 9709_w18_MS_21 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use $y$ values 2, $\sqrt{2.5}$ , 1 or equivalents | B1 | |
| | Use correct formula, or equivalent, with $h=\frac{1}{2}\pi$ and three $y$ values | M1 | |
| | Obtain $\frac{1}{2}\times\frac{1}{2}\pi(2+2\sqrt{2.5}+1)$ or equivalent and hence 4.84 | A1 | |
| | | 3 | |
| (ii) | State or imply volume is $\int\pi(1+3\cos^2\frac{1}{2}x)\,dx$ | B1 | Allow if $\pi$ appears later; condone omission of d$x$ |
| | Use appropriate identity to express integrand in form $k_1+k_2\cos x$ | M1 | |
| | Obtain $\int\pi(\frac{5}{2}+\frac{3}{2}\cos x)\,dx$ or $\int(\frac{5}{2}+\frac{3}{2}\cos x)\,dx$ | A1 | Condone omission of d$x$ |
| | Integrate to obtain $\pi(\frac{5}{2}x+\frac{3}{2}\sin x)$ or $\frac{5}{2}x+\frac{3}{2}\sin x$ | A1 | |
| | Obtain $\frac{5}{2}\pi^2$ with no errors seen | A1 | |
| | | 5 | |

167. 9709_w18_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Integrate to obtain form $k\ln(3x+2)$ | *M1 | Condone poor use of brackets if recovered later |
| | Obtain correct $4\ln(3x+2)$ | A1 | |
| | Substitute limits correctly | M1 | Dependent *M, must see $k\ln 20 - k\ln 5$ oe |
| | Apply relevant logarithm properties correctly | M1 | Dependent *M, do not allow $\frac{4\ln 20}{4\ln 5}$ oe, must be using both the subtraction and power laws correctly |
| | Obtain $\ln 256$ nfww | A1 | AG; necessary detail needed |
| | | 5 | |
| (b) | Use identity to obtain $4(1-\cos 2x)$ oe | B1 | |
| | Use identity to obtain $\sec^2 2x - 1$ | B1 | |
| | Integrate to obtain form $k_1x + k_2\sin 2x + k_3\tan 2x$ | *M1 | Allow M1 if integrand contains $p\cos 2x + q\sec^2 2x$ and no other trig terms |
| | Obtain correct $3x - 2\sin 2x + \frac{1}{2}\tan 2x$ | A1 | |
| | Apply limits correctly retaining exactness | M1 | Dependent *M, allow $\sin\frac{\pi}{3}$, $\tan\frac{\pi}{3}$ |
| | Obtain $\frac{1}{2}\pi - \frac{1}{2}\sqrt{3}$ or exact equivalent | A1 | |
| | | 6 | |

168. 9709_m17_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use $\cos(A+B)$ identity | M1 | |
| | Obtain $2\cos 2x\left(\cos 2x.\frac{1}{2}\sqrt{3} - \sin 2x.\frac{1}{2}\right)$ | A1 | |
| | Attempt identity expressing $\cos^2 2x$ in terms of $\cos 4x$ | M1 | |
| | Attempt identity expressing $\cos 2x \sin 2x$ in terms of $\sin 4x$ | M1 | |
| | Obtain $\frac{1}{2}\sqrt{3}(1+\cos 4x) - \frac{1}{2}\sin 4x$ | A1 | |
| | **Total:** | 5 | |
| (ii) | Attempt to find at least one intercept with $x$-axis | M1 | |
| | Obtain $x = \frac{1}{6}\pi$ at least | A1 | |
| | Integrate to obtain $k_4x + k_5\sin 4x + k_6\cos 4x$ | M1 | |
| | Obtain $\frac{1}{2}\sqrt{3}x + \frac{1}{8}\sqrt{3}\sin 4x + \frac{1}{8}\cos 4x$ | A1√ | following their answer to **(i)** of correct form |
| | Apply limits 0 and $\frac{1}{6}\pi$ to obtain $\left(\frac{1}{12}\sqrt{3}\right)\pi$ or exact equivalent | A1 | following completely correct work |
| | **Total:** | 5 | |

169. 9709_s17_MS_21 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| | Integrate to obtain form $ke^{\frac{1}{2}x+3}$ where $k$ is constant not equal to 4 | M1 | |
| | Obtain correct $8e^{\frac{1}{2}x+3}$ | A1 | Allow unsimplified for **A1** |
| | Obtain $8e^{\frac{1}{2}a+3}-8e^3=835$ or equivalent | A1 | |
| | Carry out correct process to find $a$ from equation of form $ke^{\frac{1}{2}a+3}=c$ | M1 | |
| | Obtain 3.65 | A1 | If 3.65 seen with no actual attempt at integration, award **B1** if it is thought that trial and improvement with calculator has been used. |
| | **Total:** | 5 | |

170. 9709_s17_MS_21 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | State or imply correct $y$-values 0, $\tan\frac{1}{6}\pi$, $\tan\frac{2}{6}\pi$ | B1 | Some candidates have their calculator in degree mode when working out $\tan\frac{\pi}{6}$ etc. this gives 0.00915 and 0.0183. Allow **B1**. |
| | Use correct formula, or equivalent, with $h=\frac{1}{12}\pi$ and $y$–values | M1 | Must be convinced they have considered 3 values for $y$ for **M1** |
| | Obtain 0.378 | A1 | |
| | **Total:** | 3 | |
| (ii) | State or imply $\pi\int\left(\sec^2 2x-1\right)\mathrm{d}x$ | B1 | |
| | Integrate to obtain $k_1\tan 2x+k_2x$, any non-zero constants including $\pi$ or not | M1 | |
| | Obtain $\frac{1}{2}\tan 2x-x$ or $\pi\left(\frac{1}{2}\tan 2x-x\right)$ | A1 | |
| | Obtain $\pi\left(\frac{1}{2}\sqrt{3}-\frac{1}{6}\pi\right)$ or equivalent | A1 | |
| | **Total:** | 4 | |

171. 9709_s17_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Obtain $\int(2\cos^2\theta - \cos\theta - 3)\,d\theta$ | B1 | |
| | Attempt use of identity to obtain integrand involving $\cos 2\theta$ and $\cos\theta$ | M1 | |
| | Integrate to obtain form $k_1 \sin 2\theta + k_2 \sin\theta + k_3\theta$ for non-zero constants | M1 | |
| | Obtain $\frac{1}{2}\sin 2\theta - \sin\theta - 2\theta + c$ | A1 | |
| | **Total:** | 4 | |
| (b)(i) | Integrate to obtain form $k_1 \ln(2x+1) + k_2 \ln(x)$ or $k_1 \ln(2x+1) + k_2 \ln(2x)$ | M1 | |
| | Obtain $2\ln(2x+1) + \frac{1}{2}\ln x$ or $2\ln(2x+1) + \frac{1}{2}\ln(2x)$ | A1 | |
| | **Total:** | 2 | |
| (b)(ii) | Use relevant logarithm power law for expression obtained from application of limits | M1 | |
| | Use relevant logarithm addition / subtraction laws | M1 | |
| | Obtain $\ln 18$ | A1 | |
| | **Total:** | 3 | |

172. 9709_w17_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Obtain integrand of form $a\sec^2\theta + b$ | M1 | |
| | Obtain correct $5\sec^2\theta - 1$ | A1 | |
| | Integrate to obtain form $a\tan\theta + b\theta$ | M1 | |
| | Obtain $5\tan\theta - \theta + c$ | A1 | |
| | | 4 | |
| (b) | Obtain integral of form $k\ln(3x+1)$ | *M1 | |
| | Apply limits and obtain $\frac{2}{3}\ln(3a+1) = \ln 16$ | A1 | |
| | Obtain equation with no presence of ln | DM1 | |
| | Obtain 21 | A1 | |
| | | 4 | |

173. 9709_w17_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Obtain $2-2\cos 2x$ as part of integrand | **B1** | |
| | Obtain $3\sin 2x$ as part of integrand | **B1** | Allow second **B1** for writing |
| | Integrate to obtain form $k_1x+k_2\sin 2x+k_3\cos 2x$ | **M1** | $\int 6\sin x\cos x\ \mathrm{d}x = 6\left(\frac{1}{2}\sin^2 x\right)$, **M1** may then be implied by subsequent work |
| | Obtain $2x-\sin 2x-\frac{3}{2}\cos 2x$ or $2x-\sin 2x+3\sin^2 x$ | **A1** | |
| | Apply limits to obtain $\frac{1}{2}\pi+\frac{1}{2}$ | **A1** | |
| | | **5** | |
| (b) | Integrate to obtain $2\ln(3x+2)$ | **B1** | Allow $\frac{6}{3}\ln(3x+2)$ for **B1** |
| | Use at least one relevant logarithm property | ***M1** | |
| | Obtain $\frac{3a+2}{2}=7$ or $\frac{(3a+2)^2}{4}=49$ or equivalent without ln | **A1** | |
| | Solve relevant equation to find $a$ | **DM1** | Dep on ***M1**, allow for $49=(3a+2)^2$ OE or correct working involving $(3a+2)$ |
| | Obtain $a=4$ only | **A1** | |
| | | **5** | |

174. 9709_m16_MS_22 Q: 5

| | | |
|---|---|---|
| Obtain integral of form $ke^{2x+1}$ | **M1** | |
| Obtain correct $3e^{2x+1}$ | **A1** | |
| Apply both limits correctly and rearrange at least to $e^{2a+1}=\ldots$ | **M1** | |
| Use logarithms correctly to find $a$ | **M1** | |
| Obtain 1.097 | **A1** | [5] |

175. 9709_m16_MS_22 Q: 8

| | | | |
|---|---|---|---|
| **(i)** | State $2\sin x\cos x \cdot \dfrac{\cos x}{\sin x}$ | **B1** | |
| | Simplify to confirm $2\cos^2 x$ | **B1** | [2] |
| **(ii) (a)** | Use $\cos 2x = 2\cos^2 x - 1$ | **B1** | |
| | Express in terms of $\cos x$ | **M1** | |
| | Obtain $16\cos^2 x + 3$ or equivalent | **A1** | |
| | State 3, following their expression of form $a\cos^2 x + b$ | **A1** | [4] |
| **(b)** | Obtain integrand as $\dfrac{1}{2}\sec^2 2x$ | **B1** | |
| | Integrate to obtain form $k\tan 2x$ | **M1*** | |
| | Obtain correct $\dfrac{1}{4}\tan 2x$ | **A1** | |
| | Apply limits correctly | **dep M1*** | |
| | Obtain $\dfrac{1}{4}\sqrt{3} - \dfrac{1}{4}$ or exact equivalent | **A1** | [5] |

176. 9709_s16_MS_21 Q: 7

| | | | |
|---|---|---|---|
| **(a)** | Rewrite integrand as $\sec^2 2x + \cos^2 2x$ | **B1** | |
| | Express $\cos^2 2x$ in form $k_1 + k_2\cos 4x$ | **M1** | |
| | State correct $\frac{1}{2} + \frac{1}{2}\cos 4x$ | **A1** | |
| | Integrate to obtain at least terms involving $\tan 2x$ and $\sin 4x$ | **M1** | |
| | Obtain $\frac{1}{2}\tan 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x$ , condoning absence of $+c$ | **A1** | [5] |
| **(b)** | Integrate to obtain $2x + 2\ln(3x-2)$ | **B1** | |
| | Show correct use of $p\ln k = \ln k^p$ law at least once, must be using $\ln(3x-2)$ | **M1** | |
| | Show correct use of $\ln m - \ln n = \ln\frac{m}{n}$ law, must be using $\ln(3x-2)$ | **M1** | |
| | Use or imply $20 = \ln(e^{20})$ | **B1** | |
| | Obtain $\ln(16e^{20})$ | **A1** | [5] |

177. 9709_s16_MS_22 Q: 6

**(a)** Obtain integrand $2e^{-2x} + \frac{1}{2}e^{-x}$ **B1**

Obtain integral of form $k_1e^{-2x} + k_2e^{-x}$ **M1**

Obtain answer $-e^{-2x} - \frac{1}{2}e^{-x}$, condoning absence of $+c$ **A1** [3]

**(b)** Integrate to obtain $\frac{1}{2}\ln(2x+5)$ **B1**

Show correct use of $p\ln k = \ln k^p$ law at least once **M1**

Show correct use of $\ln m - \ln n = \ln\frac{m}{n}$ law **M1**

Obtain $\ln\frac{5}{3}$ **A1** [4]

**(c)** State or imply correct ordinates $\log 2, \log 5, \log 8$ or decimal equivalents **B1**

Use correct formula, or equivalent, correctly with $h = 3$ and 3 ordinates **M1**

Obtain answer 3.9 with no errors seen **A1** [3]

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178. 9709_w16_MS_21 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | Use $\cos 2x = 2\cos^2 x - 1$ and attempt factorisation of numerator<br>Obtain $(2\cos x + 1)(\cos x + 4)$<br>Confirm given result $2\cos x + 1$ | **M1**<br>**A1**<br>**A1** | [3] |
| **(ii)** | Express integrand as $2\cos 2x + 1$<br>Integrate to obtain $\sin 2x + x$<br>Apply limits correctly to integral of form $k_1 \sin 2x + k_2 x$<br>Obtain $2\pi$ | **B1**<br>**B1**<br>**M1**<br>**A1** | [4] |

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179. 9709_w16_MS_22 Q: 3

| | | | | |
|---|---|---|---|---|
| **(i)** | Obtain integral of form $k_1e^{\frac{1}{2}x} + k_2x$<br>Obtain correct $8e^{\frac{1}{2}x} + 3x$ oe<br>Use limits correctly to confirm $8e - 2$ | **M1**<br>**A1**<br>**A1** | [3] | Allow $k_1 = 4$ |
| **(ii)** | Draw increasing curve in first quadrant<br>Draw more or less accurate sketch with correct curvature,<br>gradient at $x = 0$ must be >0 | **M1**<br><br>**A1** | [2] | If incorrect $y$ intercept used then M1 A0<br>Allow if no intercept stated |
| **(iii)** | State more and refer to top(s) of trapezium(s) above curve | **B1** | [1] | Can be shown using a diagram.<br>Reference to a trapezium must be made |

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180. 9709_w16_MS_22 Q: 6

| | | | | |
|---|---|---|---|---|
| (i) | Use $\cos 2\theta = 2\cos^2\theta - 1$ appropriately twice | B1 | | Alternative method $\frac{1-2\sin^2\theta}{2\cos^2\theta} = \frac{1}{2}\sec^2\theta - \tan^2\theta$ or $\frac{1}{2\cos^2\theta} - \tan^2\theta$ B1 |
| | Simplify to confirm $1 - \frac{1}{2}\sec^2\theta$ | B1 | [2] | then as for 2nd B1 |
| (ii) | Use $\sec^2\alpha = 1 + \tan^2\alpha$ | B1 | | |
| | Obtain equation $\tan^2\alpha + 10\tan\alpha + 25 = 0$ or equivalent | B1 | | |
| | Attempt solution of 3-term quadratic equation for $\tan\alpha$ and use correct process for finding value of $\alpha$ from negative value of $\tan\alpha$ | M1 | | If quadratic is incorrect, need to see evidence of attempt to solve as required to obtain M1 |
| | Obtain 1.77 | A1 | [4] | Allow better or in terms of $\pi$ $\left(\frac{1013\pi}{1800}\right)$ |
| (iii) | State or imply integrand $1 - \frac{1}{2}\sec^2\frac{1}{2}x$ | B1 | | |
| | Obtain integral of form $k_1x - k_2\tan\frac{1}{2}x$ | M1 | | |
| | Obtain correct $x - \tan\frac{1}{2}x$ | A1 | | |
| | Apply limits correctly to obtain $\pi - 2$ | A1 | [4] | |

181. 9709_w16_MS_23 Q: 3

| | | | |
|---|---|---|---|
| (i) | Rewrite integrand as $\sec^2 4x - 1$ | B1 | |
| | Integrate to obtain $\frac{1}{4}\tan 4x - x$, condoning absence of $+c$ | B1 | [2] |
| (ii) | Integrate to obtain $2\sin 2x - 2\cos 3x$ | B1 | |
| | Apply limits correctly to integral of form $k_1\sin 2x + k_2\cos 3x$ | M1 | |
| | Obtain $3 - \sqrt{2}$ or exact equivalent | A1 | [3] |

182. 9709_w16_MS_23 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | State or imply correct ordinates $\sqrt{2}$, $\sqrt{1+e}$, $\sqrt{1+e^2}$ or decimal equivalents<br>Use correct formula, or equivalent, correctly with $h = 3$ and three ordinates<br>Obtain answer 12.25 with no errors seen | **B1**<br>**M1**<br>**A1** | [3] |
| **(ii)** | Refer to top of each trapezium being above curve or equivalent | **B1** | [1] |
| **(iii)** | State or imply volume is $\int \pi(1+e^{\frac{1}{3}x})\,dx$<br>Integrate to obtain form $k_1x + k_2e^{\frac{1}{3}x}$ with or without $\pi$<br>Obtain correct $\pi(x+3e^{\frac{1}{3}x})$ or $x+3e^{\frac{1}{3}x}$<br>Obtain $\pi(3+3e^2)$ or exact equivalent | **B1**<br>**M1**<br>**A1**<br>**A1** | [4] |

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183. 9709_s15_MS_21 Q: 6

**(i)** State or imply $\operatorname{cosec} 2\theta = \dfrac{1}{\sin 2\theta}$ — B1
Express left-hand side in terms of $\sin\theta$ and $\cos\theta$ — M1
Obtain given answer $\sec^2\theta$ correctly — A1 [3]

**(ii)** **(a)** State or imply $\cos\theta = \dfrac{1}{\sqrt{5}}$ or $\tan\theta = 2$ at least — B1
Obtain 1.11 or awrt 1.11, allow $0.353\pi$ — B1
Obtain 2.03 or awrt 2.03 , allow $0.648\pi$ and no other values between 0 and $\pi$ — B1 [3]

**(b)** State integrand as $\sec^2 2x$ — B1
Integrate to obtain expression of form $k\tan mx$ — M1
Obtain correct $\frac{1}{2}\tan 2x$ — A1
Obtain $\frac{1}{2}\sqrt{3}$ or exact equivalent — A1 [4]

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184. 9709_s15_MS_22 Q: 4

**(i)** Differentiate to obtain $e^x - 8e^{-2x}$ — B1
Use correct process to solve equation of form $ae^x + be^{-2x} = 0$ — M1
Confirm given answer $\ln 2$ correctly — A1 [3]

**(ii)** Integrate to obtain expression of form $pe^x + qe^{-2x}$ — M1
Obtain correct $e^x - 2e^{-2x}$ — A1
Apply both limits correctly — M1 depM
Confirm given answer $\dfrac{5}{2}$ — A1 [4]

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185. 9709_s15_MS_22 Q: 6

| | | | |
|---|---|---|---|
| **(i)** | Solve three-term quadratic equation for $\sin x$ | M1 | |
| | Obtain at least $\sin x = -\frac{1}{2}$ and no errors seen | A1 | |
| | Obtain $x = \frac{7}{6}\pi$ | A1 | [3] |
| **(ii)** | State $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$ | B1 | |
| | Obtain given $5 + 8\sin x - 2\cos 2x$ with necessary detail seen | B1 | |
| | Integrate to obtain expression of form $ax + b\cos x + c\sin 2x$ | M1 | |
| | Obtain correct $5x - 8\cos x - \sin 2x$ | A1 | |
| | Apply limits 0 and their $x$-value correctly | M1 depM | |
| | Obtain $\frac{35}{6}\pi + \frac{7}{2}\sqrt{3} + 8$ or exact equivalent | A1 | [6] |

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186. 9709_w15_MS_21 Q: 5

| | | | |
|---|---|---|---|
| **(a)** | Use $\tan^2 x = \sec^2 x - 1$ | **B1** | |
| | Obtain integral of form $p\tan x + qx + r\cos 2x$ | **M1** | |
| | Obtain $\tan x - x - \dfrac{1}{2}\cos 2x + c$ | **A1** | [3] |
| **(b)** | Obtain integral of form $ke^{1-2x}$ | **M1*** | |
| | Obtain $-\dfrac{3}{2}e^{1-2x}$ | **A1** | |
| | Apply both limits the correct way round | **M1 dep** | |
| | Obtain $-\dfrac{3}{2}e^{-1} + \dfrac{3}{2}e$ or exact equivalent | **A1** | [4] |

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187. 9709_w15_MS_22 Q: 7

| | | | |
|---|---|---|---|
| **(i)** | Express $\cos^2 x$ in form $k_1 + k_2\cos 2x$ | **M1** | |
| | Obtain correct $\frac{1}{2} + \frac{1}{2}\cos 2x$ | **A1** | |
| | Rewrite second term as $\sec^2 x$ | **B1** | |
| | Integrate to obtain at least terms $k_3\sin 2x$ and $k_4\tan x$ | **M1** | |
| | Obtain $\frac{1}{2}x + \frac{1}{4}\sin 2x + \tan x$ | **A1** | |
| | Confirm given result $\frac{1}{6}\pi + \frac{9}{8}\sqrt{3}$ | **A1** | [6] |
| **(ii)** | State volume is $\pi\int(\cos x + \dfrac{1}{\cos x})^2$ ($\pi$ maybe implied by later appearance) | **B1** | |
| | Expand to obtain $\pi\int(\cos^2 x + \dfrac{1}{\cos^2 x} + 2)\mathrm{d}x$ or $\int(\cos^2 x + \dfrac{1}{\cos^2 x} + 2)\mathrm{d}x$ | **B1** | |
| | Integrate integrand involving three terms (in part using part (i) or otherwise i.e. $k_3\sin 2x + k_4\tan x + k_5 x$ ) | **M1** | |
| | Obtain $\frac{5}{6}\pi^2 + \frac{9}{8}\sqrt{3}\pi$ or exact equivalent | **A1** | [4] |

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188. 9709_w15_MS_23 Q: 1

| | | |
|---|---|---|
| Integrate to obtain $k\ln(2x+5)$ | **M1** | |
| Obtain correct $\frac{3}{2}\ln(2x+5)$ | **A1** | |
| Apply limits and use logarithm law for $\ln a - \ln b$ | **M1** | |
| Use logarithm power law | **M1** | |
| Obtain $\ln 125$ | **A1** | [5] |

189. 9709_w20_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Use iteration correctly at least once | **M1** | Need to see 3 values including the starting values |
| | Obtain final answer 1.817 | **A1** | Answer required to exactly 4 significant figures |
| | Show sufficient iterations to 6 significant figures to justify answer or show sign change in interval $[1.8165,\ 1.8175]$ | **A1** | |
| | | **3** | |
| (b) | State equation $x=\dfrac{6+8x}{8+x^2}$ or equivalent using $\alpha$ | **B1** | |
| | Obtain $\sqrt[3]{6}$ or exact equivalent | **B1** | |
| | | **2** | |

190. 9709_w20_MS_22 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (a) | Substitute $x=3$ and attempt evaluation | **M1** | |
| | Obtain 0 and confirm factor $x-3$ | **A1** | AG |
| | | **3** | |
| (b) | Divide quartic expression by $x-3$ at least as far as $x^3+kx^2$ | **M1** | |
| | Obtain $x^3-2x^2$ | **A1** | |
| | Obtain $x^3-2x^2+5$ | **A1** | With no errors seen |
| | Attempt rearrangement of their cubic expression to $x=\ldots$ | **M1** | Or $a=\ldots$ |
| | Confirm $a=-\sqrt{\dfrac{5}{2-a}}$ | **A1** | AG |
| | | **5** | |
| (c) | Use iteration process correctly at least once | **M1** | Need to see 3 values including *their* starting value. |
| | Obtain final answer $-1.24$ | **A1** | Answer required to exactly 3 significant figures. |
| | Show sufficient iterations to 5 sf to justify answer or show a sign change in the interval $[-1.245,\ -1.235]$ | **A1** | |
| | | **3** | |

191. 9709_m19_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Attempt rearrangement of $\frac{e^{2x}}{4x+1}=10$ to $x=\ldots$ involving ln | M1 | |
| | Confirm $x=\frac{1}{2}\ln(40x+10)$ | A1 | Answer given; necessary detail needed |
| | | 2 | |
| (ii) | Use iteration process correctly at least once | M1 | |
| | Obtain final answer 2.316 | A1 | |
| | Show sufficient iterations to 6 sf to justify answer or show a sign change in the interval [2.3155, 2.3165] | A1 | |
| | | 3 | |
| (iii) | Use quotient rule (or product rule) to find derivative | M1 | |
| | Obtain $\frac{2e^{2x}(4x+1)-4e^{2x}}{(4x+1)^2}$ or equivalent | A1 | |
| | Substitute answer from part **(ii)** (or more accurate value) into attempt at first derivative | M1 | |
| | Obtain 16.1 | A1 | |
| | | 4 | |

192. 9709_s19_MS_21 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use quotient rule (or product rule) to differentiate | M1 | Penalise missing brackets by withholding the A mark unless recovered later |
| | Obtain $\frac{dy}{dx}=\frac{3x^2(2-5x)-(-5)(8+x^3)}{(2-5x)^2}$ or equivalent | A1 | |
| | State or imply curve crosses $x$-axis when $x=-2$ | B1 | |
| | Substitute $-2$ to obtain 1 | A1 | |
| | | 4 | |
| (ii) | Equate numerator of first derivative to zero and rearrange as far as $kx^3=\ldots$ or equivalent | M1 | |
| | Confirm given result $x=\sqrt{0.6x+4x^{-1}}$ **AG** | A1 | Condone in this part error(s) in denominator of derivative |
| | | 2 | |
| (iii) | Use iterative process correctly at least once | M1 | |
| | Obtain final answer 1.81 | A1 | |
| | Show sufficient iterations to 5 sf to justify answer or show a sign change in the interval [1.805, 1.815] | A1 | |
| | | 3 | |

193. 9709_s19_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Equate $4t^2e^{-t}$ to 1, rearrange to $t^2 = \ldots$ and hence $t = \ldots$ | M1 | Allow M1 for $t=\sqrt{\frac{1}{4}e^{-t}}$ |
| | Confirm $t=\frac{1}{2}e^{\frac{1}{2}t}$ with necessary detail needed as answer is given | A1 | |
| | | 2 | |
| (ii) | Use iterative process correctly at least once | M1 | |
| | Obtain final answer $t = 0.715$ | A1 | |
| | Show sufficient iterations to 5 sf to justify answer or show a sign change in the interval [0.7145, 0.7155] | A1 | **SC**: M1A1 from iterations to 4sf resulting in 0.71 |
| | | 3 | |
| (iii) | Obtain $\frac{dx}{dt}=3+12e^{-2t}$ | B1 | |
| | Use product rule to find $\frac{dy}{dt}$ | M1 | |
| | Obtain $8te^{-t}-4t^2e^{-t}$ | A1 | |
| | Divide correctly to obtain $\frac{dy}{dx}$ | M1 | |
| | Substitute value from part **(ii)** to obtain 0.31 | A1 | Allow greater accuracy |
| | | 5 | |

194. 9709_w19_MS_21 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Integrate to obtain form $x^3+k_1\sin 2x+k_2\cos x$ | *M1 | |
| | Obtain correct $x^3+2\sin 2x+\cos x$ | A1 | |
| | Apply limits correctly and equate to 2 | DM1 | |
| | Confirm given result | A1 | AG; necessary detail needed |
| | | 4 | |
| (ii) | Consider sign of $a-\sqrt[3]{3-2\sin 2a-\cos a}$ or equivalent for 0.5 and 0.75 | M1 | |
| | Obtain –0.26 and 0.10 or equivalents and justify conclusion | A1 | AG; necessary detail needed |
| | | 2 | |
| (iii) | Use iterative process correctly at least once | M1 | Need to see a correct $x_3$, may be implied by $x_1=0.5$ so $x_3=0.65256$ or $x_1=0.75$ so $x_3=0.64897$ OE<br>Must be working with radians |
| | Obtain final answer 0.651 | A1 | |
| | Show sufficient iterations to 5sf to justify answer or show a sign change in the interval [0.6505, 0.6515] | A1 | |
| | | 3 | |

195. 9709_w19_MS_22 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use iteration correctly at least once | M1 | Must see correct attempt at $x_3$ |
| | Obtain final answer 1.359 | A1 | |
| | Show sufficient iterations to 6 sf to justify answer or show sign change in interval [1.3585, 1.3595] | A1 | Answer required to exactly 4 sf<br>Must see to at least $x_5$ |
| | | 3 | |
| (ii) | Form correct equation in $x$ (or $\alpha$ ) | B1 | $x = \frac{x}{\ln 2x}$ OE |
| | Obtain $\frac{1}{2}e$ | B1 | |
| | | 2 | |

196. 9709_m18_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Integrate to obtain $-2e^{-2x}$ | B1 | |
| | Apply limits correctly to integral of form $ke^{-2x}$ | M1 | |
| | Obtain $-2e^{-4a} + 2e^{2a} = 25$ | A1 | |
| | Rearrange to confirm $a = \frac{1}{2}\ln(12.5 + e^{-4a})$ | A1 | AG; necessary detail needed |
| | | 4 | |
| (ii) | Consider sign of $a - \frac{1}{2}\ln(12.5 + e^{-4a})$ or equivalent for 1.0 and 1.5 | M1 | |
| | Obtain $-0.26$ and 0.24 or equivalent and justify conclusion | A1 | AG; necessary detail needed |
| | | 2 | |
| (iii) | Use iterative process correctly at least once | M1 | |
| | Obtain final answer 1.263 | A1 | |
| | Show sufficient iterations to 6 sf to justify answer or show a sign change in the interval (1.2625, 1.2635) | A1 | |
| | | 3 | |

197. 9709_s18_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use quotient rule or equivalent | M1 | Obtaining two terms in numerator and $(2x+1)^2$ in denominator for a quotient |
| | Obtain correct $\frac{\frac{5}{x}(2x+1)-10\ln x}{(2x+1)^2}$ or equivalent, or $\frac{5}{x}(2x+1)^{-1}-10\ln x(2x+1)^{-2}$ or equivalent | A1 | Obtaining one term with $(2x+1)^{-1}$ oe and a second term with $(2x+1)^{-2}$ oe for a product<br>Condone poor use of brackets if recovered later |
| | Substitute $x=1$ to obtain $\frac{15}{9}$ or $\frac{5}{3}$ or equivalent, www | A1 | |
| | | 3 | |
| (ii) | Equate numerator to zero and attempt relevant arrangement | M1 | For M1, need to see at least one line of working after either $10+\frac{5}{x}-10\ln x=0$ or their numerator (which must have at least 2 terms, one involving $\ln x$) = 0 |
| | Confirm $x=\frac{x+0.5}{\ln x}$ | A1 | AG; necessary detail needed |
| | | 2 | |
| (iii) | Use iteration process correctly at least once | M1 | |
| | Obtain final answer 3.181 | A1 | |
| | Show sufficient iterations to 6 sf to justify answer or show sign change in interval $(3.1805, 3.1815)$ | A1 | |
| | | 3 | |

198. 9709_s18_MS_22 Q: 6

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Rewrite integrand as $1+2e^{\frac{1}{2}x}+e^x$ | B1 | |
| | Integrate to obtain form $x+k_1e^{\frac{1}{2}x}+k_2e^x$ | M1 | |
| | Obtain $x+4e^{\frac{1}{2}x}+e^x$ | A1 | |
| | Use limits to obtain $a+4e^{\frac{1}{2}a}+e^a-5=10$ | A1 | |
| | Rearrange as far as $e^{\frac{1}{2}a}=\ldots$ including use of $4e^{\frac{1}{2}a}+e^a=e^{\frac{1}{2}a}(4+e^{\frac{1}{2}a})$ | M1 | |
| | Confirm $a=2\ln\left(\frac{15-a}{4+e^{\frac{1}{2}a}}\right)$ | A1 | AG; necessary detail needed |
| | | 6 | |
| (ii) | Consider sign of $a-2\ln\left(\frac{15-a}{4+e^{\frac{1}{2}a}}\right)$ for 1.5 and 1.6 or equivalent | M1 | |
| | Obtain $-0.08...$ and $0.06...$ or equivalents and justify conclusion | A1 | |
| | | 2 | |
| (iii) | Use iterative process correctly at least once | M1 | |
| | Obtain final answer 1.56 | A1 | |
| | Show sufficient iterations to 5 sf to justify answer or show sign change in interval $(1.555, 1.565)$ | A1 | |
| | | 3 | |

199. 9709_w18_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Substitute –2 and simplify | M1 | |
| | Obtain $16-16+8+24-32$ and hence zero and conclude | A1 | AG; necessary detail needed |
| | | 2 | |
| (ii) | Attempt division by $x+2$ to reach at least partial quotient $x^3+kx$ or use of identity or inspection | M1 | |
| | Obtain $x^3+2x-16$ | A1 | |
| | Equate to zero and obtain $x=\sqrt[3]{16-2x}$ | A1 | |
| | | 3 | |
| (iii) | Use iteration process correctly at least once | M1 | |
| | Obtain final answer 2.256 | A1 | |
| | Show sufficient iterations to 6 sf to justify answer or show a sign change in the interval (2.2555, 2.2565) | A1 | |
| | | 3 | |

200. 9709_w18_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Rearrange at least as far as $2x=\ln(...)$ | M1 | Allow if in terms of $p$, need to see $y$ equated to 0 |
| | Obtain $x=\frac{1}{2}\ln(1.6x^2+4)$ | A1 | AG; necessary detail needed |
| | | 2 | |
| (ii) | Either | | |
| | Consider sign of $x-\frac{1}{2}\ln(1.6x^2+4)$ for 0.75 and 0.85 or equivalent | M1 | Need to see substitution of numbers |
| | Obtain –0.04 and 0.03 or equivalents and justify conclusion | A1 | AG; necessary detail needed, change of sign or equivalent must be mentioned |
| | Or | | |
| | Consider sign of $5e^{2x}-8x^2-20$ for 0.75 and 0.85 | M1 | Need to see substitution of numbers |
| | Obtain –2.09... and 1.58... or equivalents and justify conclusion | A1 | AG; necessary detail needed, change of sign or equivalent must be mentioned |
| | | 2 | |
| (iii) | Use iteration process correctly at least once | M1 | Starting with value such that iterations converge to correct values |
| | Obtain final value 0.80956 | A1 | Must be 5sf for the final answer |
| | Show sufficient iterations to justify value or show sign change in interval (0.809555, 0.809565) | A1 | |
| | | 3 | |
| (iv) | Obtain first derivative $10e^{2x}-16x$ | B1 | |
| | Substitute value from iteration to find gradient, must be in the form $pe^{2x}+qx$ | M1 | |
| | Obtain 37.5 | A1 | Or greater accuracy, allow awrt 37.5 from use of $x=0.8096,\ 0.80955$ oe |
| | | 3 | |

201. 9709_m17_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Integrate to obtain form $k_1x + k_2x^2 + k_3e^{3x}$ for non-zero constants | M1 | |
| | Obtain $x + x^2 + e^{3x}$ | A1 | |
| | Apply both limits to obtain $a + a^2 + e^{3a} - 1 = 250$ or equivalent | A1 | |
| | Apply correct process to reach form without e involved | M1 | |
| | Confirm given $a = \frac{1}{3}\ln(251 - a - a^2)$ | A1 | |
| | **Total:** | 5 | |
| (ii) | Use iterative process correctly at least once | M1 | |
| | Obtain final answer 1.835 | A1 | |
| | Show sufficient iterations to 6 sf to justify answer or show sign change in interval (1.8345, 1.8355) | A1 | |
| | **Total:** | 3 | |

202. 9709_s17_MS_21 Q: 4

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Use iteration correctly at least once | M1 | |
| | Obtain final answer 2.08 | A1 | |
| | Show sufficient iterations to 4 dp to justify answer or show sign change in interval (2.075, 2.085) | A1 | |
| | **Total:** | 3 | |
| (ii) | State or clearly imply equation $x = \frac{2x^2 + x + 9}{(x+1)^2}$ or same equation using $\alpha$ | B1 | |
| | Carry out relevant simplification | M1 | |
| | Obtain $\sqrt[3]{9}$ | A1 | |
| | | 3 | |

203. 9709_s17_MS_22 Q: 3

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Draw sketch of $y = x^3$ | *B1 | May be implied by part graph in first quadrant |
| | Draw straight line with negative gradient crossing positive $y$-axis and indicate one intersection | DB1 | dep *B |
| | **Total:** | 2 | |
| (ii) | Use iterative formula correctly at least once | M1 | |
| | Obtain final answer 1.926 | A1 | |
| | Show sufficient iterations to justify 4 sf or show sign change in interval $(1.9255, 1.9265)$ | A1 | |
| | **Total:** | 3 | |

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204. 9709_w17_MS_21 Q: 7

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Differentiate to obtain form $k_1x + k_2 + k_3 \sin\frac{1}{2}x$ | *M1 | |
| | Obtain correct $2x + 3 - \frac{5}{2}\sin\frac{1}{2}x$ and deduce or imply gradient at $P$ is 3 | A1 | |
| | Equate first derivative to their $-3$ and rearrange | DM1 | |
| | Obtain $x = \frac{5}{4}\sin\frac{1}{2}x - 3$ | A1 | |
| | | 4 | |
| (ii) | Consider sign of their $2x + 6 - \frac{5}{2}\sin\frac{1}{2}x$ at $-4.5$ and $-4.0$ or equivalent | M1 | |
| | Complete argument correctly for correct expression with appropriate calculations | A1 | |
| | | 2 | |
| (iii) | Use iteration formula correctly at least once | M1 | |
| | Obtain final answer $-4.11$ | A1 | |
| | Show sufficient iterations to justify accuracy to 3 sf or show sign change in interval $(-4.115, -4.105)$ | A1 | |
| | | 3 | |

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205. 9709_w17_MS_22 Q: 5

| | Answer | Mark | Partial Marks |
|---|---|---|---|
| (i) | Obtain derivative of the form $ke^{-2x}$ | *M1 | Condone $k = 4$ for **M1** |
| | State or imply gradient of curve at $P$ is –8 | A1 | |
| | Form equation of straight line through $(0, 9)$ with negative gradient | *DM1 | dep on *M |
| | Obtain $y = -8x + 9$ or equivalent | A1 | |
| | Equate equation of curve and equation of straight line | DM1 | dep on both *M |
| | Rearrange to confirm $x = \frac{9}{8} - \frac{1}{2}e^{-2x}$ | A1 | |
| | | **6** | |
| (ii) | Use iterative process correctly at least once | M1 | |
| | Obtain final answer 1.07 | A1 | |
| | Show sufficient iterations to 5 sf to justify answer or show sign change in interval $(1.065, 1.075)$ | A1 | |
| | | **6** | |

206. 9709_m16_MS_22 Q: 4

**(i)** Use the iterative formula correctly at least once — **M1**
Obtain final answer 1.516 — **A1**
Show sufficient iterations to justify accuracy to 3 dp or show sign change in interval $(1.5155, 1.5165)$ — **B1** [3]

**(ii)** State equation $x = \sqrt{\frac{1}{2}x^2 + 4x^{-3}}$ or equivalent — **B1**
Obtain exact value $\sqrt[5]{8}$ or $8^{0.2}$ — **B1** [2]

207. 9709_s16_MS_21 Q: 6

| | | | |
|---|---|---|---|
| **(i)** | Use quotient rule or equivalent | ***M1** | |
| | Obtain $\frac{6x(x^2+4)-6x^3}{(x^2+4)^2}$ or equivalent | **A1** | |
| | Equate first derivative to $\frac{1}{2}$ and remove algebraic denominators dep on *M1 | **DM1** | |
| | Obtain $48p = p^4+8p^2+16$ or $48x = x^4+8x^2+16$ or equivalent | **A1** | |
| | Confirm given result $p=\sqrt{\frac{48p-16}{p^2+8}}$ | **A1** | [5] |
| **(ii)** | Consider sign of $p-\sqrt{\frac{48p-16}{p^2+8}}$ at 2 and 3 or equivalent | **M1** | |
| | Complete argument correctly with appropriate calculations | **A1** | [2] |
| **(iii)** | Carry out iteration process correctly at least once | **M1** | |
| | Obtain final answer 2.728 | **A1** | |
| | Show sufficient iterations to justify accuracy to 4 sf or show sign change in interval (2.7275, 2.7285) | **B1** | [3] |

208. 9709_s16_MS_22 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | Use product rule to obtain form $k_1e^{\frac{1}{3}x}+k_2xe^{\frac{1}{3}x}$ | ***M1** | |
| | Obtain correct $6e^{\frac{1}{3}x}+2xe^{\frac{1}{3}x}$ | **A1** | |
| | Equate first derivative to 40 and obtain equation without e present, dep *M | **DM1** | |
| | Confirm $p=3\ln\frac{20}{p+3}$ or $x=3\ln\frac{20}{x+3}$ | **A1** | [4] |
| **(ii)** | Consider sign of $p-3\ln\frac{20}{p+3}$ at 3.3 and 3.5 or equivalent | **M1** | |
| | Complete argument correctly with appropriate calculations | **A1** | [2] |
| **(iii)** | Carry out iterative process correctly at least once | **M1** | |
| | Obtain final answer 3.412 | **A1** | |
| | Show sufficient iterations to justify accuracy to 3 dp or show sign change in interval (3.4115, 3.4125) | **B1** | [3] |

209. 9709_w16_MS_21 Q: 4

| | | | |
|---|---|---|---|
| **(i)** | Integrate to obtain $2e^{2x}+5x$<br>Apply limits correctly and equate to 100<br>Rearrange and apply logarithms correctly to reach $a=...$<br>Confirm given result $a=\frac{1}{2}\ln(50+e^{-2a}-5a)$ | **B1**<br>**M1**<br>**M1**<br>**A1** | [4] |
| **(ii)** | Use the iterative formula correctly at least once<br>Obtain final answer 1.854<br>Show sufficient iterations to justify accuracy to 3 dp or show sign change in interval (1.8535, 1.8545) | **M1**<br>**A1**<br>**B1** | [3] |

210. 9709_w16_MS_22 Q: 5

| | | | | |
|---|---|---|---|---|
| (i) | Use quotient rule (or product rule) to find first derivative | M1 | | Quotient: Must have a difference in the numerator and $(x^2+1)^2$ in the denominator |
| | Obtain $\dfrac{\frac{4}{x}(x^2+1)-8x\ln x}{(x^2+1)^2}$ or equivalent | A1 | | Product: Must see an application of the chain rule. |
| | State $\frac{4}{x}(x^2+1)-8x\ln x=0$ or equivalent | A1 | | Condone missing brackets if correct use is implied by correct work later |
| | Carry out correct process to produce equation without ln, without any incorrect working | M1 | | |
| | Confirm $m=e^{0.5(1+m^{-2})}$ or $x=e^{0.5(1+x^{-2})}$ | A1 | [5] | |
| (ii) | Use iterative formula correctly at least once | M1 | | Should not be attempting to use $x_0=0$, but if used and 'recovered' then SC M1 A1- usually see $m_1=1.6487$ |
| | Obtain final answer 1.895 | A1 | | |
| | Show sufficient iterations to 6 sf to justify answer or show sign change in interval $(1.8945, 1.8955)$ | A1 | [3] | |

211. 9709_w16_MS_23 Q: 1

| | | | |
|---|---|---|---|
| (i) | Use the iterative formula correctly at least once<br>Obtain final answer 2.289<br>Show sufficient iterations to justify accuracy to 3 dp or show sign change in interval (2.2885, 2.2895) | M1<br>A1<br>B1 | [3] |
| (ii) | State equation $x=\dfrac{4}{x^2}+\frac{2}{3}x$ or equivalent<br>Obtain exact value $12^{\frac{1}{3}}$ or $\sqrt[3]{12}$ | B1<br>B1 | [2] |

212. 9709_s15_MS_21 Q: 5

(i) Obtain integral of form $ke^{\frac{1}{2}x}+mx$ — M1

Obtain correct $6e^{\frac{1}{2}x}+x$ — A1

Apply limits and obtain correct $6e^{\frac{1}{2}a}+a-6$ — A1

Equate to 10 and introduce natural logarithm correctly — DM1

Obtain given answer $a=2\ln\left(\frac{16-a}{6}\right)$ correctly — A1 [5]

(ii) Use the iterative formula correctly at least once — M1

Obtain final answer 1.732 — A1

Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (1.7315, 1.7325) — A1 [3]

213. 9709_s15_MS_22 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | Draw recognisable sketch of $y = 16 - x^4$ | B1 | |
| | Draw recognisable sketch of $y = \|3x\|$ | B1 | |
| | Indicate in some way the two points of intersection | B1 depBB | [3] |
| **(ii)** | Use iterative process correctly at least once | M1 | |
| | Obtain final answer 1.804 | A1 | |
| | Show sufficient iterations to justify answer or show sign change in the interval (1.8035, 1.8045) | A1 | [3] |
| **(iii)** | State (1.804, 5.412) | B1 | |
| | State (−1.804, 5.412), following their first point | B1√ | [2] |

214. 9709_w15_MS_21 Q: 4

| | | | |
|---|---|---|---|
| **(i)** | Make a recognisable sketch of $y = \ln x$ | **B1** | |
| | Draw straight line with negative gradient crossing positive *y*-axis and justify one real root | **B1** | [2] |
| **(ii)** | Consider sign of $\ln x + \frac{1}{2}x - 4$ at 4.5 and 5.0 or equivalent | **M1** | |
| | Complete the argument correctly with appropriate calculations | **A1** | [2] |
| **(iii)** | Use the iterative formula correctly at least once | **M1** | |
| | Obtain final answer 4.84 | **A1** | |
| | Show sufficient iterations to justify accuracy to 2 d.p. or show sign change in interval (4.835, 4.845) | **A1** | [3] |

215. 9709_w15_MS_22 Q: 2

| | | | |
|---|---|---|---|
| **(i)** | Use the iterative formula correctly at least once | **M1** | |
| | Obtain final answer 2.289 | **A1** | |
| | Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (2.2885, 2.2895) | **A1** | [3] |
| **(ii)** | State $x = 2 + \frac{4}{x^2 + 2x + 4}$ or equivalent | **B1** | |
| | Obtain $\sqrt[3]{12}$ | **B1** | [2] |

216. 9709_w15_MS_23 Q: 5

| | | | |
|---|---|---|---|
| **(i)** | Integrate to obtain $e^{3x} + 5e^x$ | **B1** | |
| | Apply both limits and subtract for expression of form $k_1e^{3x} + k_2e^x$ | **M1** | |
| | Obtain $e^{3a} + 5e^a = 106$ or similarly simplified equivalent | **A1** | |
| | Rearrange and introduce logarithms | **M1** | |
| | Confirm given answer $a = \frac{1}{3}\ln(106 - 5e^a)$ | **A1** | [5] |
| **(ii)** | Use the iterative formula correctly at least once | **M1** | |
| | Obtain final answer 1.477 | **A1** | |
| | Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (1.4765, 1.4775) | **A1** | [3] |